Price Stabilization and the Risk-Averse Firm

Robert G. Chambers[1] and John Quiggin[2]

May 15, 2003

[1]Professor of Agricultural and Resource Economics, University of Maryland and Adjunct Professor of Agricultural and Resource Economics, University of Western Australia.

[2]Australian Research Council Senior Fellow, Australian National University.

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This paper develops a firm-level welfare analysis of two types (complete and partial) of mean-preserving price stabilization for producers with general risk-averse preferences facing a stochastic technology represented by a state-contingent production correspondence. Following Sandmo, producer preferences are defined over *ex post* profit or net returns. Special cases of the model that follows include models based upon expected-utility preferences and a stochastic production function, models based upon mean-variance preferences and a stochastic production function, models based upon generalized expected utility preferences and a stochastic production function, the preceding preferences with non-stochastic production, as well as risk-neutral behavior. Hence, at the firm level, the model can reasonably be characterized as encompassing the cases considered in the existing Sandmovian literature.

The primary goal of the paper is to develop an analytically simple and tractable approach to the welfare analysis of price stabilization for risk-averse firms facing a stochastic price and production environment. The crucial analytical tool used in achieving this goal is the indirect objective function of the firm capturing the interaction between technology and preferences.

In what follows, we first present our model and notation. After that, we formally define our notion of price stabilization. It includes as special cases among others, the notions of price stabilization used by Sandmo, Quiggin and Anderson (1979, 1981)Eeckhoudt and Hansen, Holt, and Finkelshtain and Chalfant. Because we are interested in results that cover the entire spectrum of aversion to risk, we consider two special cases of preferences that bracket the range of preferences for risk-averse individuals, risk-neutral preferences and completely risk-averse preferences. We consider general risk-averse preferences in the penultimate section, and the final section concludes. Results developed for a number of special cases, including those analyzed by Oi, Sandmo, Blandford and Currie, Quiggin and Anderson (1979, 1981), Eeckhoudt and Hansen, and others, can be derived as corollaries of our general results.

The Model and Notation

Uncertainty is modelled by 'Nature' making a choice from a finite set of states $\Omega = \{1, 2, ..., S\}$. We denote by $\mathbf{1}$ the $S-$dimension vector of ones. The state-contingent production technology, following Chambers and Quiggin (2000), is modelled by a continuous input correspondence, $X : \Re_{+}^{S} \rightarrow \Re_{+}^{N}$, which maps vectors of state-contingent outputs, $\mathbf{z}$, into inputs capable of producing them

$$X(\mathbf{z}) = \left\{\mathbf{x} \in \Re_{+}^{N} : \mathbf{x} \text{ can produce } \mathbf{z}\right\} \quad \mathbf{z} \in \Re_{+}^{S}.$$

The scalar $z_s \in \Re_{+}$ denotes the *ex post* or realized output in state s. To maximize comparability with the existing literature, we only consider the case of a scalar stochastic output. In addition to continuity, the input correspondence satisfies:[1]

X.1 $X(0_S) = \Re_{+}^{N}$, and $\mathbf{0}_N \notin X(\mathbf{z})$ for $\mathbf{z} \geq \mathbf{0}_S$ and $\mathbf{z} \neq \mathbf{0}_S$.

X.2 $\mathbf{z}' \geq \mathbf{z} \Rightarrow X(\mathbf{z}) \subseteq X(\mathbf{z}')$.

X.3 $\lambda X(\mathbf{z}^0) + (1-\lambda) X(\mathbf{z}^1) \subseteq X(\lambda \mathbf{z}^0 + (1-\lambda)\mathbf{z}^1) \quad 0 < \lambda < 1,$

Individual producers face stochastic output prices, $\mathbf{p} \in \Re_{++}^{S}$, and non-stochastic input prices, $\mathbf{w} \in \Re_{++}^{N}$. Producers have no ability to affect either the stochastic output prices they face or the input prices. Their preferences are defined over *ex post* income,[2] $\mathbf{y} \in \Re^{S}$,

$$y_s = p_s z_s - \mathbf{wx}.$$

Their evaluation of these *ex post* incomes is given by a continuous and nondecreasing certainty equivalent function, $e : \Re^{S} \rightarrow \Re$ satisfying Aczél's agreement property

$$e(\mu \mathbf{1}) = \mu, \mu \in \Re.$$

By standard duality theorems (Färe, 1988), there is a cost function dual to $X(\mathbf{z})$ and defined

$$c(\mathbf{w}, \mathbf{z}) = \min\left\{\mathbf{wx} : \mathbf{x} \in X(\mathbf{z})\right\}$$

if $X(\mathbf{z})$ is nonempty and ∞ otherwise. The cost function satisfies:

C.1. $c(\mathbf{w}, \mathbf{z})$ is positively linearly homogeneous, non-decreasing, concave, and continuous in $\mathbf{w}$;

C.2. Shephard's Lemma;

C.3. $c(\mathbf{w}, \mathbf{z}) \geq 0$, $c(\mathbf{w}, 0_S) = 0$, and $c(\mathbf{w}, \mathbf{z}) > 0$ for $\mathbf{z} \geq 0_S, \mathbf{z} \neq 0_S$;

C.4. $c(\mathbf{w}, \mathbf{z})$ is convex and continuous on $\Re^S_{++}$.

For analytic convenience, we typically strengthen C.4 to permit differentiability of the cost structure.

The *indirect certainty equivalent* (ICE) is defined

$$I(\mathbf{w}, \mathbf{p}) = \sup_{\mathbf{z}} \{e(\mathbf{pz} - c(\mathbf{w}, \mathbf{z})\mathbf{1})\}. \tag{1}$$

Chambers and Quiggin (2001) prove that under these assumptions on the technology, $I(\mathbf{w}, \mathbf{p})$ is continuous in $(\mathbf{w}, \mathbf{p})$, nondecreasing in $\mathbf{p}$, and nonincreasing and quasi-convex in $\mathbf{w}$. For convenience we shall typically strengthen those properties to include differentiability in $\mathbf{p}$ and assume that there exists a unique solution to (**??**), which we refer to as the *state-contingent supply vector* and denote by

$$\mathbf{z}(\mathbf{w}, \mathbf{p}) = \arg\max_{\mathbf{z}} \{e(\mathbf{pz} - c(\mathbf{w}, \mathbf{z})\mathbf{1})\}.$$

Generally, one expects that risk-averse individuals will, to some degree, try to balance price uncertainty against production uncertainty in an attempt to smooth their income distribution. Accordingly, we say that an individual *self insures in the neighborhood of* $\mathbf{p}$ if for all s and k

$$[z_s(\mathbf{w}, \mathbf{p}) - z_k(\mathbf{w}, \mathbf{p})](p_s - p_k) \leq 0. \tag{2}$$

Self insurance requires that there be a negative correlation between state-contingent prices and supplies. If there is a perfect inverse correlation between prices and state-contingent incomes, income is nonstochastic. The individual has *completely self insured.* As shown below, complete self insurance can emerge under appropriate assumptions on the producer's risk aversion.

Given a vector of probabilities $\boldsymbol{\pi}$, a function $f : \Re^S \to \Re$ is said to be *generalized Schur concave* if $\mathbf{v} \preceq_{\pi} \mathbf{v}'$ implies $f(\mathbf{v}) \geq f(\mathbf{v}')$, where the notation $\mathbf{v} \preceq_{\pi} \mathbf{v}'$ means that $\mathbf{v}$ and $\mathbf{v}'$ have the same mean and $\mathbf{v}$ is less risky than $\mathbf{v}'$ in the sense of Rothschild and Stiglitz (Chambers and Quiggin, 1997). A function, f, is *generalized Schur convex* if $-f$ is generalized Schur concave. Examples of functions which are generalized Schur concave include the risk-averse expected utility functional, the risk-neutral expected utility function, the linear mean-variance preference function, linear mean-standard deviation, and maximin, or *completely risk averse*, preferences

$$e(\mathbf{y}) = \min\{y_1, ..., y_S\}.$$

Maximin preferences are generalized Schur concave for all possible probability vectors. A function is differentiable and generalized Schur concave if and only if

$$\left(\frac{f_s(\mathbf{v})}{\pi_s} - \frac{f_t(\mathbf{v})}{\pi_t}\right)(v_s - v_t) \leq 0, \tag{3}$$

for all s, t where subscripts on functions denote partial derivatives (Marshall and Olkin; Chambers and Quiggin, 1997).

Defining Price Stabilization and Assessing its Consequences

In this section, we take perhaps two obvious steps. We formally define our notion of price stabilization, and we introduce a general criterion for price stabilization to be welfare improving.

The indirect certainty equivalent provides an ideal vehicle by which to examine the welfare effects of price stabilization. To state the obvious, consider two price distributions $\mathbf{p}^0$ and $\mathbf{p}^1$. Then if $\mathbf{p}^1$ is more stable, in some sense, than $\mathbf{p}^0$, the producer gains from the more stable prices if and only if

$$I\left(\mathbf{w}, \mathbf{p}^1\right) \geq I\left(\mathbf{w}, \mathbf{p}^0\right).$$

To make this definition operational and meaningful, a definition of what it means for $\mathbf{p}^0$ to be more stable than $\mathbf{p}^1$ is needed. Many studies (e.g., Finkelshtain and Chalfant, Sandmo) have focused on the case of stabilization at the mean of the price distribution:

$$\mathbf{p}^1 = \left(\sum_{s\in\Omega} \pi_s p_s^0\right)\mathbf{1}.$$

Here $\boldsymbol{\pi} \in \Re^S_{++}$ is an objectively defined vector of probabilities. We refer to stabilization at the mean of the distribution as *complete* stabilization.

Given a set of objectively defined probabilities, one can easily conceive of situations where one price distribution, which is not degenerately stochastic, is more stable than another price distribution. Figure 1 illustrates. There, the line passing through $\mathbf{p}^0$ has slope given by the relative probabilities of the two states of Nature (the fair-odds). Complete stabilization of $\mathbf{p}^0$ is illustrated by the degenerate price distribution defined by the intersection between the fair-odds line and the 45^0 ray emanating from the origin. (We refer to the 45^0 ray as the *bisector*.) Price distributions lying on the same fair-odds line as $\mathbf{p}^0$ but closer to the bisector are more stable than the latter in the sense that they have the same mean, but their distribution is less spread out.

Figure 2 illustrates the difference between the cumulative probability distributions for $\mathbf{p}^1$ and $\mathbf{p}^0$. Both have the same mean, but the latter (solid line) has more weight in the lower tail than the former. This is what we will take as our operating definition of a more stable price distribution. More formally, we will say that $\mathbf{p}^1$ is *more stable* than $\mathbf{p}^0$ given the probability vector $\boldsymbol{\pi}$, denoted $\mathbf{p}^1 \preceq_{\pi} \mathbf{p}^0$, if they have the same mean and $\mathbf{p}^1$ is less risky than $\mathbf{p}^0$ in the sense of Rothschild and Stiglitz. Because the degenerate distribution with the mean occuring with probability one dominates all probability distributions in the Rothschild-Stiglitz sense, complete stabilization is an important polar case.

The type of price-brand stabilization considered theoretically by Eeckhoudt and Hansen and empirically by Holt as well as others represents a special case of this type of stabilization. Our definition of price stabilization, however, does not encompass several traditional methods of price stabilization in the United States and Europe. For example, establishing minimum guaranteed prices through either a support-price or target-price mechanism truncates the lower tail of the price distribution faced by producers. Hence, such schemes are mean enhancing and not mean preserving, and thus inevitably involve implicit price subsidization as well as price stabilization. We leave the consideration of mean-enhancing price stabilization mechanisms to future work.

With this definitiion of stabilization in hand, it then follows that the producer benefits from a more stable price distribution if and only if

$$\mathbf{p}^1 \preceq_\pi \mathbf{p}^0 \Rightarrow I\left(\mathbf{w},\mathbf{p}^1\right) \geq I\left(\mathbf{w},\mathbf{p}^0\right).$$

This observation leads to our first result:

Proposition 1 *If $I(\mathbf{w},\mathbf{p})$ is generalized Schur concave (convex), then the producer always benefits (always loses) from a more stable price distribution.*

One case where this proposition obviously applies is the Sandmovian model, where production is non-stochastic, and preferences are of the expected-utility form. More generally, notice that if production is nonstochastic, then $c(\mathbf{w},\mathbf{z})$ degenerates to the usual single-output, nonstochastic cost function, which we denote by $c^c(\mathbf{w},z)$, where z is now a scalar non-stochastic output. Clearly,

$$\mathbf{p}^1 \preceq_\pi \mathbf{p}^0 \Rightarrow z\left(\mathbf{w},\mathbf{p}^0\right)\mathbf{p}^1 - c^c\left(\mathbf{w},z\left(\mathbf{w},\mathbf{p}^0\right)\right)\mathbf{1} \preceq_\pi z\left(\mathbf{w},\mathbf{p}^0\right)\mathbf{p}^0 - c^c\left(\mathbf{w},z\left(\mathbf{w},\mathbf{p}^0\right)\right)\mathbf{1}.$$

Thus, for arbitrary generalized Schur concave preferences

$$\begin{aligned} I\left(\mathbf{w},\mathbf{p}^1\right) &\geq e\left(z\left(\mathbf{w},\mathbf{p}^0\right)\mathbf{p}^1 - c^c\left(\mathbf{w},z\left(\mathbf{w},\mathbf{p}^0\right)\right)\mathbf{1}\right) \\ &\geq e\left(z\left(\mathbf{w},\mathbf{p}^0\right)\mathbf{p}^0 - c^c\left(\mathbf{w},z\left(\mathbf{w},\mathbf{p}^0\right)\right)\mathbf{1}\right) \\ &= I\left(\mathbf{w},\mathbf{p}^0\right), \end{aligned}$$

where the first inequality follows by the optimality of $z(\mathbf{w},\mathbf{p}^1)$ for price distribution $\mathbf{p}^1$.

Corollary 2 *If preferences are generalized Schur concave and production is nonstochastic, the producer always benefits from price stabilization.*

Corollary **??** extends the original Sandmovian result to the entire class of generalized Schur concave preferences. These include among others such non expected utility models as linear mean-standard deviation preferences, completely risk averse preferences, and rank-dependent expected utility models as well as the risk-averse expected utility class of preference functionals. Iit is difficult to establish general conditions under which $I(\mathbf{w},\mathbf{p})$ is generalized Schur concave globally when production is stochastic because the producer's arbitrage activities across states of Nature, in the quest of self insurance, routinely counterbalance price and production risk. Hence, the operational content of Proposition **??** is limited for stochastic production structures.

However, the observation that generalized Schur concavity, in a local sense, is the underlying determinant of welfare gains and losses from price stabilization or not is important because it yields an operational method for determining whether the producer benefits from stabilization. Any price distribution $\mathbf{p}^1$ such that $\mathbf{p}^1 \preceq_\pi \mathbf{p}^0$ can be constructed from $\mathbf{p}^0$ by a sequence of mean preserving pairwise contractions of $\mathbf{p}^0$ (Marshall and Olkin). Hence, in determining whether the producer benefits or loses from price stabilization, it suffices to restrict attention to mean-preserving pairwise contractions of $\mathbf{p}^0$. This basic observation, drawn from the literature on majorization and inequality measurement, when coupled with basic duality relationships provides a foundation for much of what follows.

To illustrate, take a price distribution, $\mathbf{p}^0$, and choose two indexes s and k such that $[p_s^0 - p_k^0] > 0$. Now consider, moving from $\mathbf{p}^0$ to a more stable price distribution $\mathbf{p}^1$ by slightly decreasing p_s^0 and increasing p_k^0 in a mean preserving manner, i.e.,

$$\delta p_s^0 = -\frac{\pi_k}{\pi_s}\delta p_k^0$$

with $\delta p_k^0 > 0$ but arbitarily small. The distribution that results from this pairwise contraction is more stable than the original distribution and the resulting welfare change is proportional to

$$\left[\frac{I_k(\mathbf{w}, \mathbf{p}^0)}{\pi_k} - \frac{I_s(\mathbf{w}, \mathbf{p}^0)}{\pi_s}\right]\delta p_k^0. \tag{4}$$

Hence, the producer only gains from the more stable distribution if $\frac{I_k(\mathbf{w},\mathbf{p}^0)}{\pi_k} - \frac{I_s(\mathbf{w},\mathbf{p}^0)}{\pi_s} > 0$, and by the choice of indexes

$$\left[\frac{I_s(\mathbf{w}, \mathbf{p}^0)}{\pi_s} - \frac{I_k(\mathbf{w}, \mathbf{p}^0)}{\pi_k}\right]\left(p_s^0 - p_k^0\right) \leq 0. \tag{5}$$

By (**??**), functions satisfying (**??**) over their entire domains are generalized Schur concave.

The type of stabilization considered by Eeckhoudt and Hansen and others, which involves lowering the upper bound of the price distribution and raising the lower bound, is the special case where the two indexes are chosen so that s corresponds to the highest price in the $\mathbf{p}^0$ distribution and k corresponds to the lowest price. For this type of stabilization, often referred to as *price-band stabilization*, the local pairwise comparison yields global results.

Risk-neutral Preferences

We first consider the polar reference case of risk-neutral preferences

$$e(\mathbf{y}) = \sum_s \pi_s y_s.$$

Producers prefer complete stabilization of prices at a mean price of $\bar{p}$ if and only if there exists no price distribution $\hat{\mathbf{p}}$ such that

$$\bar{p}\mathbf{1} \preceq_\pi \hat{\mathbf{p}} \text{ and } I(\mathbf{w}, \hat{\mathbf{p}}) \geq I(\mathbf{w}, \bar{p}\mathbf{1}).$$

This last condition can be satisfied only if the producer chooses to produce a completely non-stochastic production vector when prices are stabilized. To understand why, consider any small mean preserving pairwise change of $\mathbf{p}$ in the neighborhood of $\bar{p}\mathbf{1}$. Using (**??**), the producer's welfare change is proportional to

$$\left[\frac{I_k(\mathbf{w}, \bar{p}\mathbf{1})}{\pi_k} - \frac{I_s(\mathbf{w}, \bar{p}\mathbf{1})}{\pi_s}\right] \delta p_k^0,$$

which applying the envelope theorem to (**??**) in the case of risk-neutral preferences gives

$$[z_k(\mathbf{w}, \bar{p}\mathbf{1}) - z_s(\mathbf{w}, \bar{p}\mathbf{1})]\, \delta p_k^0.$$

Hence, if the producer does not stabilize production in the face of stabilized prices, there always exists a feasible, mean-preserving, destabilizing price change that raises welfare.

Lemma 3 *If a risk-neutral producer does not stabilize production in the presence of completely stabilized prices, there always exists a beneficial departure from complete price stabilisation.*

Generally, we do not expect producers facing no price uncertainty, but production uncertainty, to stabilize production. Hence, a natural conclusion is that, generally speaking, risk-neutral producers will never prefer complete price stabilization to some price uncertainty. In the presence of a non-stochastic price p, an optimal interior solution for a risk-neutral producer must satisfy

$$\pi_s p - c_s(\mathbf{w},\mathbf{z}) = 0 \quad \forall p \tag{6}$$

Chambers and Quiggin (1997, 2000) have identified a nontrivial class of stochastic technologies for which risk-neutral producers facing no price uncertainty always stabilize production. It is the class of generalized Schur convex cost structures. For generalized Schur convex cost structures (**??**) will hold at $z\mathbf{1}$. More generally, if the cost structure is generalized Schur convex in a neighbourhood of $z\mathbf{1}$, where z is such that

$$p - \sum_s c_s(\mathbf{w}, z\mathbf{1})$$

production will be stabilized when prices are stabilized. The converse is also true. Suppose therefore that (**??**) holds and consider any price distribution $\mathbf{p}$ with $\sum_{s\in\Omega} \pi_s p_s = \bar{p}$. We have (proof in the Appendix):

Lemma 4 *If (**??**) holds and* $\mathbf{p}$ *is stochastic price vector with expected price* $\bar{p}$*, the optimal output vector* $\mathbf{z}$ *will be stochastic.*

Combining the lemmas, we conclude:

Proposition 5 *No risk-neutral producer prefers complete stabilization for all price distributions*

We now turn our attention to partial stabilization. Consider moving from $\mathbf{p}^0$ to a more stable price distribution $\mathbf{p}^1$ by making a small pairwise mean preserving change of p_k and p_s, assuming without loss of generality that $p_s^0 > p_k^0$. Applying (**??**) and using the envelope theorem in (**??**) in the case of risk-neutral preferences shows that the producer gains from this form of price stabilization if and only if

$$\left[z_k\left(\mathbf{w},\mathbf{p}^0\right) - z_s\left(\mathbf{w},\mathbf{p}^0\right)\right]\left(p_s^0 - p_k^0\right) \geq 0. \tag{7}$$

Hence, a risk-neutral producer gains if she self insures in a neighborhood of $\mathbf{p}^0$. More generally a risk-neutral producer gains from partial stabilization if there exists two states of nature sastifying (**??**).

Proposition 6 *A risk-neutral producer who self insures in a neighborhood of* $\mathbf{p}^0$ *gains from partial price stabilization.*

Because a risk-neutral producer is indifferent to income risk,if she self insures, it is as a consequence of a risk-free responses to the stochastic technology she faces. She does not self insure because of risk concerns, but rather as a result of comparing the rate at which state-contingent outputs substitute for one another in the state-contingent technology to her marginal returns in each state of Nature. This arbitraging behavior across states of Nature can lead a risk-neutral firm to self insure in the neighborhood of some price distributions. In other words, the optimal reponse to the technology outweighs her natural indifference to risk, and leads her to self insure.

Some more insight can be gathered yb using the first-order conditions to substitute out the state-contingent output prices in (**??**) to obtain

$$\left[z_k\left(\mathbf{w},\mathbf{p}^0\right) - z_s\left(\mathbf{w},\mathbf{p}^0\right)\right]\left[\frac{c_s\left(\mathbf{w},\mathbf{z}\left(\mathbf{w},\mathbf{p}^0\right)\right)}{\pi_s} - \frac{c_k\left(\mathbf{w},\mathbf{z}\left(\mathbf{w},\mathbf{p}^0\right)\right)}{\pi_k}\right] \geq 0. \tag{8}$$

It is now apparent that for a risk-neutral individual to self insure, she must choose a state-contingent supply vector that would be in the *risk aversely efficient set* in the absence of price uncertainty (Peleg and Yaari; Chambers and Quiggin, 2000). As defined by Peleg and Yaari and extended by Chambers and Quiggin (2000), the risk aversely efficient set consists of all the state-contingent supplies that might be rationally picked by an individual with generalized Schur concave preferences. As demonstrated in Chambers and Quiggin (2000), the defining structural characteristic of the risk aversely efficient set is expression (**??**).

Generally, there is no reason to expect an arbitrary price vector to be distributed in this fashion, and thus in general one does not expect a risk-neutral individual to gain from partial price stabilization. Notice, in particular, that (**??**) implies that generalized Schur convex technologies satisfy

$$\left(\frac{c_s(\mathbf{w},\mathbf{z})}{\pi_s}-\frac{c_k(\mathbf{w},\mathbf{z})}{\pi_k}\right)(z_k-z_s)\leq 0$$

for all s and k. Hence, risk-neutral individuals facing a Generalized Schur convex cost structure can never strongly satisfy (**??**),[3] and, therefore, we conclude.

Proposition 7 *A risk-neutral producer who faces a Generalized Schur convex cost structure does not gain from partial stabilization.*

The Oi Result

A classic result from nonstochastic economic theory is Oi's famous result on the desirability of price instability derived from his analysis of the supply curve. At this point, it is perhaps worthwhile to emphasize that when production is stochastic, the notion of a supply curve has to be interpreted with a caution. Well-defined notions of state-contingent supply curves exist, they are simply the graph of the state-contingent supply function over p_s. But this curve gives the way in which output varies with p_s *provided that state s occurs. It does not give a stable supply function over the entire price distribution.* If one wants to be able to depict a supply curve that gives the response of output to the price distribution, there seem to be several alternatives. One alternative is to obtain a vector of realized supplies (say from cross-section data), graph them against the observed supply prices, and then 'connect the dots'. This certainly represents a schedule of supply. However, because each observed supply corresponds to an *ex post* realization from the vector of optimal state-contingent supplies, each of which generally depends upon the entire $\mathbf{p}$ distribution, there is no reason to expect that this supply curve slopes upward or that it has any of the theoretical properties that permit the welfare evaluation of different schemes for price stabilization.

A simple, familiar example illustrates. Suppose that stochastic production is characterized by a stochastic production function, $g(\mathbf{x},\varepsilon_s)$, where as usual ε_s is a random scalar representing the random elements affecting production that are beyond the producer's control. Choose indexes, without loss of generality, so that the realization of these random elements (say across a particular cross section) are ranked from 'worst' to 'best'

$$\varepsilon_1 < ... < \varepsilon_s,$$

and assume, also as usual, that the production function is increasing in these random elements. It follows immediately as a consequence of this construction that *regardless of the distribution of* $\mathbf{p}$, holding input use fixed, supply is higher in states with higher indexes and lower in states with lower indexes. Because the rank ordering of state-contingent outputs, holding input use fixed, is determined by the rank ordering of these stochastic elements and not by the producer's allocation of inputs, there is no reason, apart from fortuitous accidents, to expect the connect-the-dots approach to yield a positively sloped supply curve in this case. If $s > k$ and $p_s < p_k$, this supply curve is negatively sloped over that region regardless of how the two producers allocated the fixed input bundle. In fact, if one expects any regularity at all to emerge, one intuitively expects a negatively sloped curve as good production states, *ceteris paribus,* would correspond to low price states.

Hence, we pursue another alternative, which is to suppose that there exists a supply mapping $S : \Re_+ \to \Re_+$ such that, for all p_s

$$z_s(\mathbf{p}, \mathbf{w}) = S(p_s, \mathbf{w})$$

If this condition is satisfied, we say that the producer's supply curve is *state-independent.* Obviously, this limits the range of technologies that can be considered. Our next lemma, which follows from applying a standard duality mapping to the ICE under the assumption of risk neutrality, identifies that class of state-contingent technologies:

Lemma 8 *A risk-neutral producer has a state-independent supply curve if and only if the cost function is of the additively separable Generalized Schur convex form:*

$$c(\mathbf{w}, \mathbf{z}) = \sum_s \pi_s \hat{c}(\mathbf{w}, z_s)$$

where $c : \Re_+ \to \Re_+$ is a convex non-stochastic cost function.

An immediate consequence of this lemma is that proposition **??** is the generalization of the Oi result to generalized Schur convex cost structures which are not additively separable.

Maximin Preferences

We now consider the polar case of complete aversion to risk given by the certainty equivalent

$$e(\mathbf{y}) = Min\{y_1, ..., y_S\}.$$

Chambers and Quiggin (2000, 2001) have established that producers facing this type of preferences and a strictly increasing cost structure completely self insure and, thus, have indirect preferences that are characterized by a *sure profit function* of the form[4]

$$I(\mathbf{w}, \mathbf{p}) = \sup_r \left\{ r - c\left(\mathbf{w}, \frac{r}{p_1}, ..., \frac{r}{p_S}\right) \right\}. \tag{9}$$

We first consider whether this most risk-averse class of producers would prefer complete stabilization at the mean price. As before, the condition that is required for the producer to prefer complete stabilization here is that there exists no price distribution $\hat{\mathbf{p}}$ such that

$$\bar{p}\mathbf{1} \preceq_\pi \hat{\mathbf{p}} \text{ and } I(\mathbf{w}, \hat{\mathbf{p}}) \geq I(\mathbf{w}, \bar{p}\mathbf{1}).$$

Consider a small pairwise spread around $\bar{p}\mathbf{1}$. Applying the envelope theorem to $I(\mathbf{w}, \bar{p}\mathbf{1})$ as defined in (**??**) establishes that the associated welfare change is proportional to

$$\left[\frac{c_k(\mathbf{w}, \mathbf{z}(\mathbf{w}, \bar{p}\mathbf{1}))}{\pi_k} - \frac{c_s(\mathbf{w}, \mathbf{z}(\mathbf{w}, \bar{p}\mathbf{1}))}{\pi_s}\right] \delta p_k.$$

So long as

$$\frac{c_k(\mathbf{w}, \mathbf{z}(\mathbf{w}, \bar{p}\mathbf{1}))}{\pi_k} \neq \frac{c_s(\mathbf{w}, \mathbf{z}(\mathbf{w}, \bar{p}\mathbf{1}))}{\pi_s},$$

for any k and s, there exists a feasible destabilization of prices that leads to a welfare improvement.

For there to be no potential for improving welfare by a small departure from stable prices, it must, therefore, be true that

$$\frac{c_k(\mathbf{w}, \mathbf{z}(\mathbf{w}, \bar{p}\mathbf{1}))}{\pi_k} = \frac{c_s(\mathbf{w}, \mathbf{z}(\mathbf{w}, \bar{p}\mathbf{1}))}{\pi_s}$$

for all k and s. Recalling (**??**) now establishes that a producer with maximin preferences prefers complete stabilization of prices over small departures from stabilization if and only if in the absence of price uncertainty she chooses a state-contingent output vector that is on the expansion path for a risk-neutral producer facing no price uncertainty. But this requires that there exist a price p such that

$$\pi_k p = c_k\left(\mathbf{w}, \mathbf{z}\left(\mathbf{w}, \bar{p}\mathbf{1}\right)\right).$$

Summing these conditions establishes

$$p - \sum_s c_s\left(\mathbf{w}, \mathbf{z}\left(\mathbf{w}, \bar{p}\mathbf{1}\right)\right) = 0.$$

The producer's first-order condition for an interior maximum for (??) for perfectly stabilized prices requires

$$\bar{p} - \sum_s c_s\left(\mathbf{w}, \mathbf{z}\left(\mathbf{w}, \bar{p}\mathbf{1}\right)\right) = 0.$$

Combining these equalities establishes that $p = \bar{p}$, or in other words the producer must choose exactly the same state-contingent output vector as a risk-neutral individual facing $\bar{p}$. Thus,

Proposition 9 *A producer with completely risk averse preferences prefers complete price stabilization over small departures from complete stabilization if and only if in the absence of price uncertainty she makes exactly the same production choices as a risk-neutral individual.*

Proposition **??** shows what appears to be a somewhat paradoxical result. That is, the most risk-averse producers generally prefer some price uncertainty to complete stabilization. However, it is easily explained. The most risk averse producers, if given the flexibility to do so, always choose their state-contingent output vector to completely self-insure. That is there will exist a perfect inverse correlation between their state-contingent output choices and state-contingent output prices. When that price uncertainty is removed, they would also want to remove all of their production uncertainty as well, and given the flexibility to do so, they would. Their interest, as with all risk averters, is in income risk and not price or production risk by themselves. Because we start at a point where prices and hence state-contingent supplies are equalized, the mean-preserving price change leads to no change in revenue, but the associated change in the state-contingent output mix which is proportional to the relative probabilites will permit the producer to lower cost, and hence increase sure profit, unless the relative probabilities are proportional to the state-contingent rate of transformation.

It may seem implausible that there exist classes of technologies for which a completely risk averse individual would produce in the same manner as a risk-neutral individual in the face of stabilized prices. However, such a class can be found in those technologies with generalized Schur convex cost structures. Chambers and Quiggin (1997, 2000) have shown that in the absence of price uncertainty, a risk-neutral individual facing a generalized Schur-convex cost structure produces the nonstochastic output level which maximizes sure profit. Hence,

Corollary 10 *A completely risk-averse individual facing a generalized Schur convex cost structure always prefers completely stabilized prices over small departures from complete stabilization.*

Because a nonstochastic production technology is a polar case of a generalized Schur convex cost structure, we also conclude as a further corollary:[5]

Corollary 11 *A completely risk-averse individual facing a nonstochastic production technology always prefers completely stabilized prices over small departures from complete stabilization.*

Now consider, moving from $\mathbf{p}^0$ to a more stable price distribution $\mathbf{p}^1$ by making a small pairwise mean preserving change of p_k and p_s, assuming without loss of generality that $p_s^0 > p_k^0$. Applying (**??**) locally to (**??**) shows that the producer with maximin preferences gains from this form of price stabilization if and only if

$$\left[\frac{c_k\left(\mathbf{w},\mathbf{z}\left(\mathbf{w},\mathbf{p}^0\right)\right)}{p_k^2\pi_k}-\frac{c_s\left(\mathbf{w},\mathbf{z}\left(\mathbf{w},\mathbf{p}^0\right)\right)}{p_s^2\pi_s}\right]\geq 0.$$

which requires

$$\frac{c_k\left(\mathbf{w},\mathbf{z}\left(\mathbf{w},\mathbf{p}^0\right)\right)/\pi_k}{c_s\left(\mathbf{w},\mathbf{z}\left(\mathbf{w},\mathbf{p}^0\right)\right)/\pi_s}>\frac{p_k^2}{p_s^2}\leq 1$$

at the producer equilibrium. Hence, a sufficient condition for a producer with maximin preferences to gain from partial stabilization is that

$$\frac{c_k\left(\mathbf{w},\mathbf{z}\left(\mathbf{w},\mathbf{p}^0\right)\right)/\pi_k}{c_s\left(\mathbf{w},\mathbf{z}\left(\mathbf{w},\mathbf{p}^0\right)\right)/\pi_s}\geq 1.$$

Because $p_s^0 > p_k^0$ implies for these preferences that $z_k\left(\mathbf{w},\mathbf{p}^0\right) - z_s\left(\mathbf{w},\mathbf{p}^0\right) > 0$, it follows that the producer gains from partial stabilization if

$$\left(\frac{c_k\left(\mathbf{w},\mathbf{z}\left(\mathbf{w},\mathbf{p}^0\right)\right)}{\pi_k}-\frac{c_s\left(\mathbf{w},\mathbf{z}\left(\mathbf{w},\mathbf{p}^0\right)\right)}{\pi_s}\right)\left(z_k\left(\mathbf{w},\mathbf{p}^0\right)-z_s\left(\mathbf{w},\mathbf{p}^0\right)\right)\geq 0. \tag{10}$$

Hence,

Proposition 12 *A producer with maximin preferences gains from partial stabilization if he faces a generalized Schur convex cost structure.*

We generalize this result in the next section to generalized Schur concave preferences.

Generalized Schur-concave Preferences

As noted earlier, both risk neutrality and complete aversion to risk are consistent with generalized Schur concavity of the preference structure. In fact, as Chambers and Quiggin (2002) demonstrate, risk neutrality and complete aversion to risk are the polar cases of preferences which are risk-averse in the sense of Yaari. Thus, they offer convenient benchmarks from which to speculate about what happens for general risk averse preference structures and technologies. Proposition **??** demonstrates that risk-neutral producers benefit from partial stabilization of prices if the stochastic technology they face leads them, for a given price distribution, to choose a state-contingent supply vector that is consistent with self insurance. In the preceding section, we have just seen that completely risk averse producers, who optimally completely self insure, will gain from partial price stabilization if they face a generalized Schur convex cost structure.

Our first result in this section generalizes these observations about risk-neutral and completely risk averse individuals and self insurance to the case of smooth generalized Schur concave preferences to show that a simple correlation condition is sufficient to determine whether a producer gains or loses from partial stabilization (proof in the Appendix).

Proposition 13 *Suppose that the producer has a smooth generalized Schur concave certainty equivalent and that at an optimal solution*

$$(z_s - z_t)(p_s z_s - p_t z_t) \leq 0,$$

all s and t. If the producer optimally self insures, he always benefits from partial stabilization of prices. If the producer does not self insure, there always exists a destabilization of prices which improves the producer's welfare.

The maintained hypothesis of Proposition **??** requires that there exist a negative correlation between the state-contingent revenues and the state-contingent supplies. If this condition is satisfied, then whether the producer gains or loses from price stabilization depends entirely upon his self insuring behavior. To understand why this condition is critical in linking self insurance and gains from stabilization, notice that it implies that if the producer self insures, then his revenue is positively correlated with the stochastic output price, i.e.,

$$(p_s - p_t)(p_s z_s - p_t z_t) \geq 0,$$

all s and t. Now suppose that the individual does self insure. Then, *ceteris paribus,* any small pairwise contraction of the price distribution raises mean income by attaching a higher price to a high production state and a lower price to a low production state. If the inverse correlation condition is satisfied and state-contingent revenue is positively correlated with the stochastic price, it also reduces the riskiness of the stochastic revenue distribution by raising revenue in the low revenue state and decreasing it in the high revenue state. Hence, even without any adjustment on her part, the producer receives a higher expected return with less risk from the pairwise contraction. She must be better off. If the producer does not self insure, then just the opposite intuition applies, destabilization leads to a higher mean and less risky revenue vector, and the risk-averse producer is thus worse off.

Notice that Proposition **??** requires that there be an inverse correlation between state-contingent supply and state-contingent revenue. This is done for the sake of concreteness in stating results. It's an easy corollary to Proposition **??** that if

$$(z_s - z_t)(p_s z_s - p_t z_t) \leq 0,$$

for any s and t, the welfare effect of a mean-preserving pairwise contraction of the price distribution thus hinges upon whether the producer self insures for those two states of Nature. We leave the exact statement of the result to the reader. It follows immediately that local stabilization can be either welfare enhancing or welfare reducing under more general conditions than laid out in Proposition **??**.

Our next task is to show that the maintained hypothesis in Proposition **??** is not vacuous in the sense that it will apply for an important class of models, i.e., producers with generalized Schur concave preferences and generalized Schur convex cost structures. Generalized Schur convex cost structures are of particular interest because as pointed out by Chambers and Quiggin (2000), they are an appropriate generalization of nonstochastic production models in a state-contingent framework. They provide cost advantages for producing a nonstochastic state contingent output, but they do not force the producer to produce nonstochastically if it is not to his advantage. In particular, suppose that the technology is generalized Schur convex for all possible probability vectors,[6]

$$c(\mathbf{w}, \mathbf{z}) = \hat{c}(\mathbf{w}, max_s \{z_1, ..., z_S\}).$$

With this cost structure and strictly monotonic certainty equivalent, a rational producer will always choose a non-stochastic output of the form $z\mathbf{1}$.[7] Hence, this class of model encompasses those used in many previous studies of stabilization under uncertainty such as those of Blandford and Currie and Quiggin and Anderson (1979, 1981).

We have

Proposition 14 *If the certainty equivalent is generalized Schur concave and the cost structure is generalized Schur convex, then at an interior solution*

$$(z_s - z_t)(p_s z_s - p_t z_t) \leq 0,$$

for all s and t.

Proposition ?? and Proposition ?? together imply the following generalization of Proposition ??

Corollary 15 *Suppose the certainty equivalent is generalized Schur concave and the cost structure is generalized Schur convex. If the producer optimally self insures, he always benefits from partial stabilization of prices. If the producer does not self insure, there always exists a destabilization of prices which improves the producer's welfare.*

Concluding comments

The purpose of the present paper has been to present a systematic treatment of the welfare effects of partial or complete mean-preserving price stabilization at the firm level. In future work, a similar approach will be applied to the analysis of supply response, mean-enchancing price stabilization, and to the stabilization of market prices using buffer stocks.

Proofs Not in Text

Proof of Lemma ??: We first prove the result for the case $S = 2$. Assume without loss of generality that for some $\delta > 0$

$$\begin{aligned} p_1 &= p - \delta \\ p_2 &= p + \frac{\pi_1}{\pi_2}\delta \end{aligned}$$

Now define the following perturbation in the optimal state-contingent supply vector $\mathbf{z}(\lambda)$

$$\begin{aligned} z_1(\lambda) &= z - \frac{\pi_2}{\pi_1}\lambda \\ z_2(\lambda) &= z + \lambda, \end{aligned}$$

$\lambda > 0$. The associated welfare perturbation for a risk-neutral individual is:

$$\delta\lambda - \lambda\frac{\pi_2}{\pi_1}c_1(\mathbf{w},z\mathbf{1}) + \lambda c_2(\mathbf{w},z\mathbf{1}) = \delta\lambda > 0$$

by (??). For the general case, observe that $p\mathbf{1} \preceq_\pi \mathbf{p}$, and therefore $p\mathbf{1}$ can be constructed from $\mathbf{p}$ by a sequence of mean preserving pairwise contractions.

Proof of Lemma ??: If: Under the stated condition,

$$\begin{aligned} I(\mathbf{w},\mathbf{p}) &= \max\left\{\sum_s \pi_s p_s z_s - \sum_s \pi_s \hat{c}(\mathbf{w}, z_s)\right\} \\ &= \sum_s \pi_s \max\{p_s z_s - \hat{c}(\mathbf{w}, z_s)\} \\ &= \sum_s \pi_s v(p_s, \mathbf{w}), \end{aligned}$$

with $v(p_s, \mathbf{w})$. convex. State-independence then follows by an application of the envelope theorem.

Only if: By the envelope theorem,

$$I_s(\mathbf{w},\mathbf{p}) = \pi_s S(p_s, \mathbf{w}),$$

so that $I(\mathbf{w},\mathbf{p})$ must be additively separable in $\mathbf{p}$. Moreover, from this expression and the convexity of I in $\mathbf{p}$ it follows that

$$\left(\frac{I_s(\mathbf{w},\mathbf{p})}{\pi_s} - \frac{I_k(\mathbf{w},\mathbf{p})}{\pi_k}\right)(p_s - p_k) \geq 0.$$

Hence, $I(\mathbf{w},\mathbf{p})$ must be additively separable and generalized Schur convex in $\mathbf{p}$,. whence

$$I(\mathbf{w},\mathbf{p}) = \sum_s \pi_s v(p_s,\mathbf{w}),$$

with v convex. By duality,

$$\begin{aligned} c(\mathbf{w},\mathbf{z}) &= \sup_p \left\{ \sum_s \pi_s p_s z_s - \sum_s \pi_s v(p_s,\mathbf{w}) \right\} \\ &= \sum_s \pi_s \sup_p \{ p_s z_s - v(p_s,\mathbf{w}) \} \\ &= \sum_s \pi_s \hat{c}(\mathbf{w}, z_s). \end{aligned}$$

Proof of Proposition 13: Applying the envelope theorem to (**??**) gives for a pairwise contraction of $\mathbf{p}^0$ (again choosing indexes as before without loss of generality so that $p_s^0 - p_k^0 > 0$)

$$\left[\frac{e_k z_k(\mathbf{w},\mathbf{p}^0)}{\pi_k} - \frac{e_s z_s(\mathbf{w},\mathbf{p}^0)}{\pi_s} \right] \delta p_k^0. \tag{11}$$

Suppose the producer self insures so that $(z_s - z_k) \leq 0$ and thus by assumption $(p_s z_s - p_k z_k) \geq 0$. Generalized Schur concavity of the preference function then ensures by (**??**)

$$\frac{e_s}{\pi_s} - \frac{e_k}{\pi_k} \leq 0,$$

which along with the fact the producer self insures, $z_s - z_k \leq 0$, then establishes

$$\frac{e_s z_s(\mathbf{w},\mathbf{p}^0)}{\pi_s} - \frac{e_k z_k(\mathbf{w},\mathbf{p}^0)}{\pi_k} \leq 0,$$

demonstrating that the self insuring producer gains from this pairwise contraction.

Now suppose that the producer does not self insure. There must, therefore, exist at the optimum at least one pair of states of Nature such that

$$\left(p_s^0 - p_k^0\right)\left(z_s\left(\mathbf{w},\mathbf{p}^0\right) - z_k\left(\mathbf{w},\mathbf{p}^0\right)\right) > 0,$$

and by assumption $p_s^0 z_s - p_k^0 z_k \leq 0$ (again choosing indexes without loss of generality so $p_s^0 - p_k^0 > 0$). Now consider a mean-preserving contraction of the price distribution $\mathbf{p}^0$ involving these two prices. Generalized Schur concavity of the certainty equivalent and the fact that $p_s^0 z_s - p_k^0 z_k \leq 0$ imply

$$\frac{e_k}{\pi_k} - \frac{e_s}{\pi_s} \leq 0,$$

and then recognizing that $z_s(\mathbf{w}, \mathbf{p}^0) - z_k(\mathbf{w}, \mathbf{p}^0) > 0$ gives the conclusion.

Proof of Proposition ??: Assume without loss of generality that $p_s > p_t$. Now suppose that $(p_s z_s - p_t z_t) \leq 0$. By the generalized Schur concavity of preferences (**??**), it then follows that

$$\frac{e_s}{\pi_s} \geq \frac{e_t}{\pi_t},$$

and the first-order conditions for an interior solution require for each z_s

$$p_s e_s - c_s \sum_t e_t = 0.$$

Combining the inequality with these equalities implies

$$\frac{c_s}{p_s \pi_s} \geq \frac{c_t}{p_t \pi_t},$$

whence by the fact that that $p_s > p_t$,

$$\frac{c_s}{\pi_s} > \frac{c_t}{\pi_t},$$

and generalized Schur convexity of the cost structure then implies

$$z_s - z_t \geq 0.$$

Supposing that $p_s > p_t$ but that $(p_s z_s - p_t z_t) > 0$ and proceeding in a similar fashion establishes that $z_s - z_t \leq 0$.

Notes

[1]These properties are discussed in detail in Chambers and Quiggin (2000, Chapter 2). Note, in particular, that they correspond to standard properties placed on input correspondences associated with nonstochastic technologies (Färe, 1988). They also generalize the representation of production uncertainty usually considered in the literature.

[2]This 'net returns' objective function is used in much of the literature on price stabilization. However, Newbery and Stiglitz (1981) and writers drawing on their work use a 'separable effort' objective function. These objective functions are discussed by Chambers and Quiggin (2000). The two coincide in the case of risk-neutral and maximin preferences or, more generally, the case of constant absolute risk aversion.

[3]Chambers and Quiggin (1997, 2000) show that this lack of self insurance implies that state-contingent supplies satisfy what they refer to as a probabilistic law of supply.

[4]If both costs and preferences are not strictly monotonic, it is easy to show that the producer can be indifferent between situations where production is economically efficient and situations where production is not economically efficient in the following sense. The producer can choose to produce at points, where because of the weak monotonicity of preferences, he would forego the chance to costlessly raise some state-contingent outputs. In such cases, general results on stabilization are much harder to state than when costs are strictly monotonic. Hence, when considering this class of weakly monotonic preferences, we restrict attention to strictly monotonic cost structures.

[5]Because completely risk-averse preferences are generalized Schur concave, Corollary ?? is also a corollary to Proposition 1.

[6]The reader will recognize that this production structure is the mirror image of the completely risk-averse preference structure.

[7]It is interesting to note that an individual with weakly increasing certainty equivalent, such as characterizes maximin preferences, risk concerns may lead them to choose a technically inefficient production point to introduce enough instability in state-contingent output to perfectly balance the price variation. A person with maximin preferences always behaves in this manner.