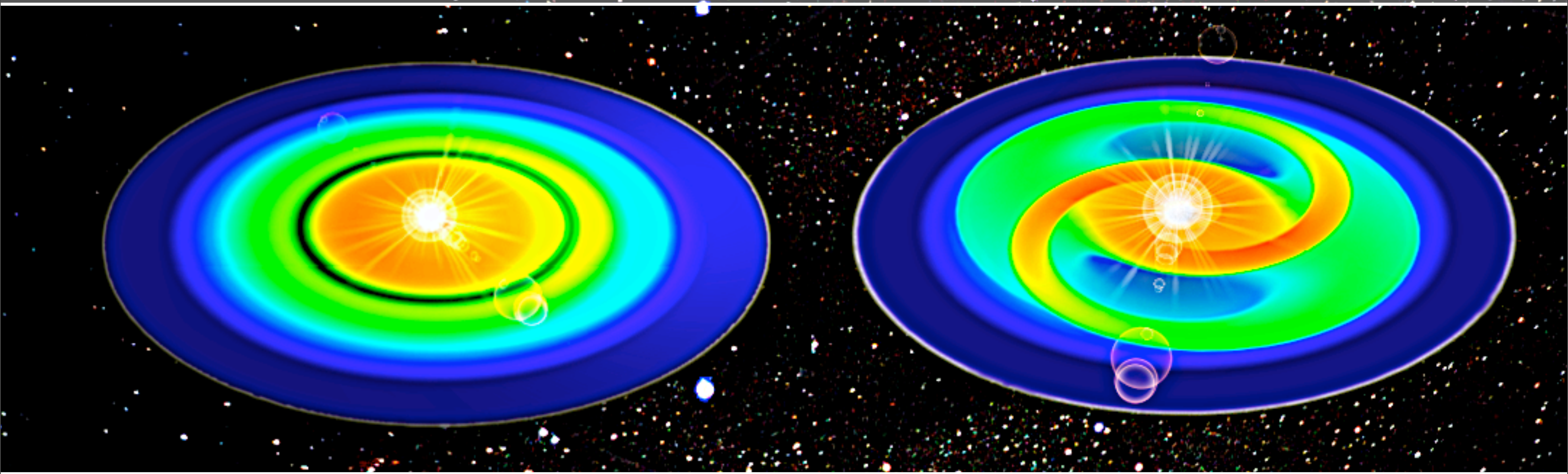
Groove modes in self-gravitating protoplanetary disks

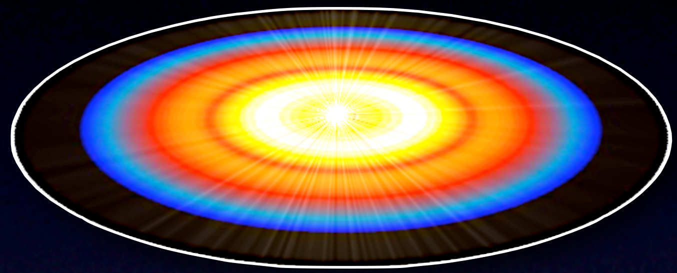


Stefano Meschiari, Greg Laughlin

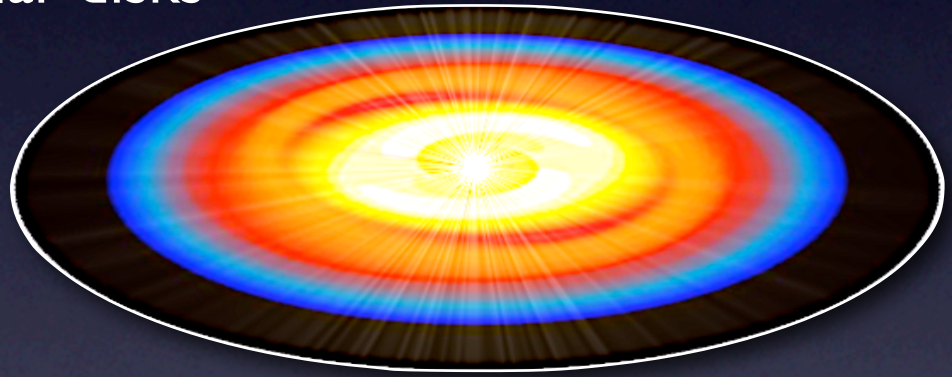
2nd year project FLASH - 07/21/2008

Meschiari and Laughlin, ApJL 2008, submitted

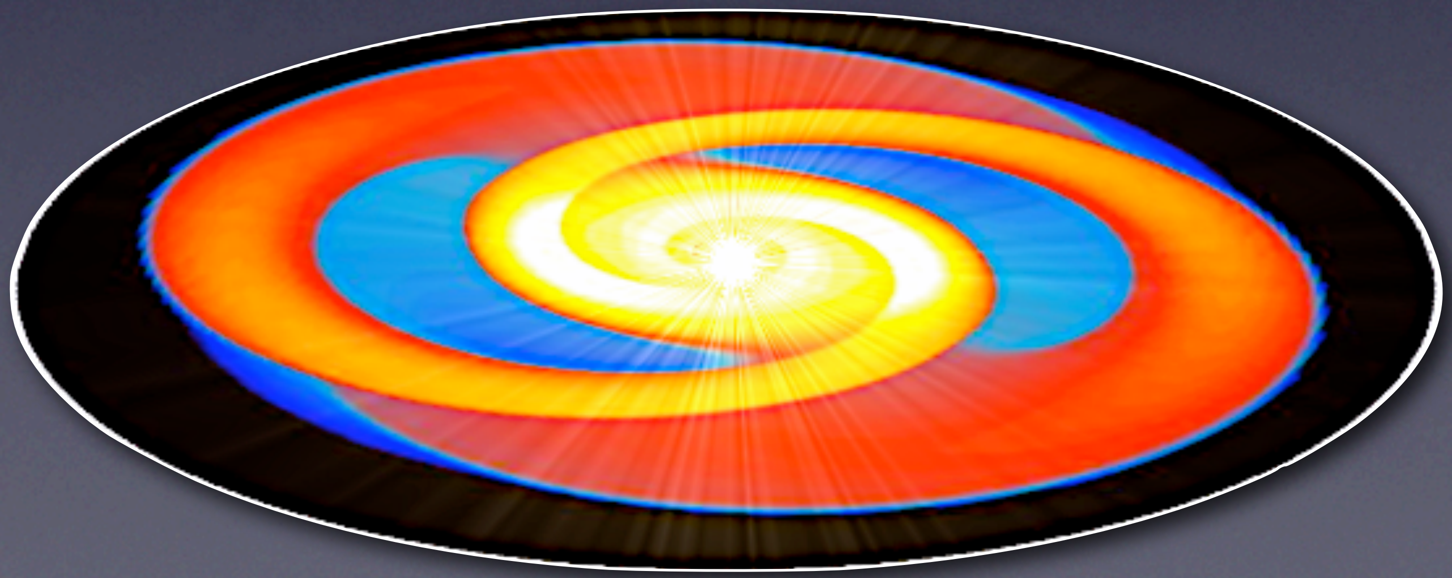
Outline



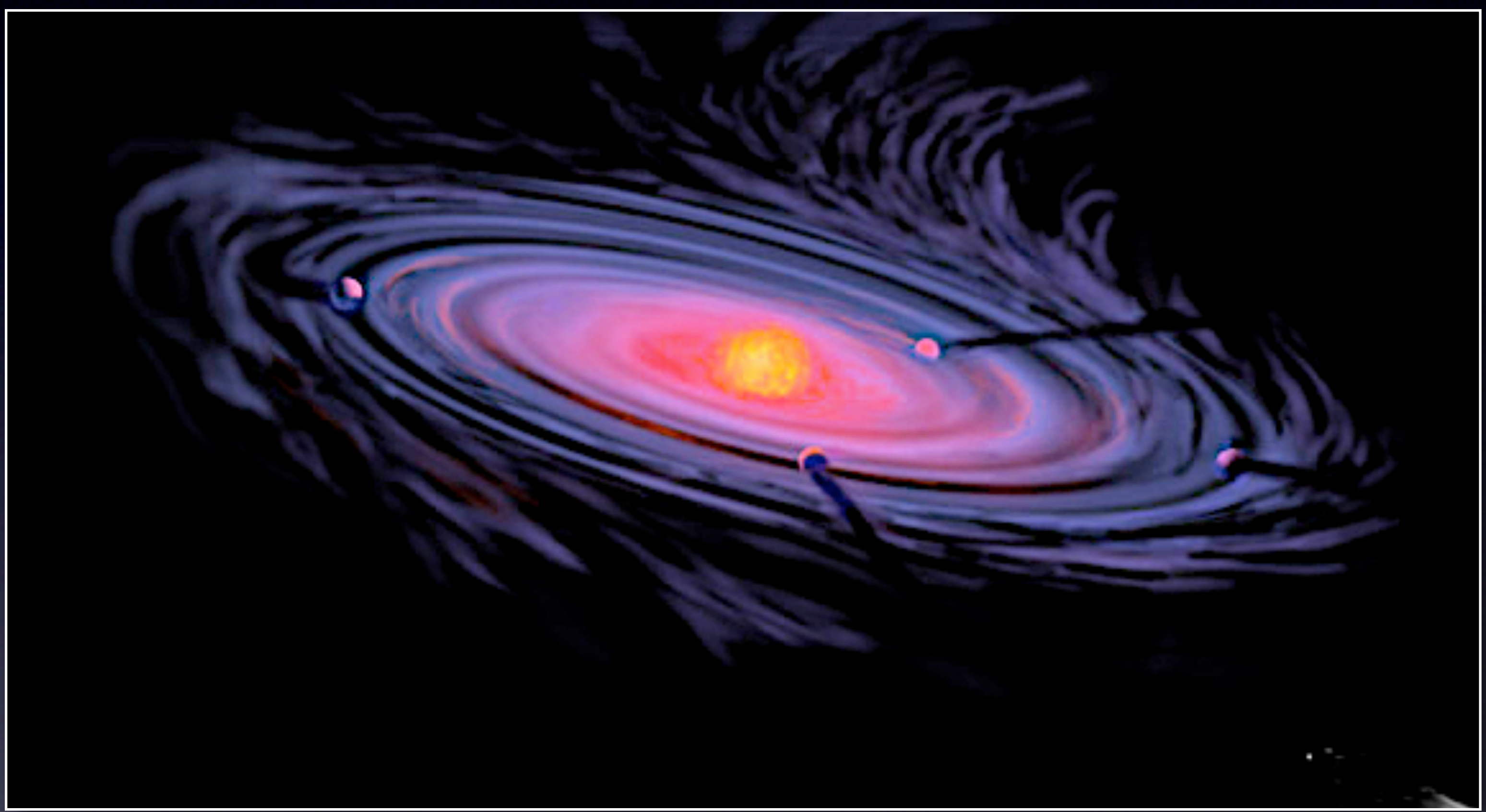
- Introduction & Motivation
  Planetary migration, gap opening, groove modes in stellar disks



- Methods
  Linear analysis, hydro simulations
- Results
- Conclusions

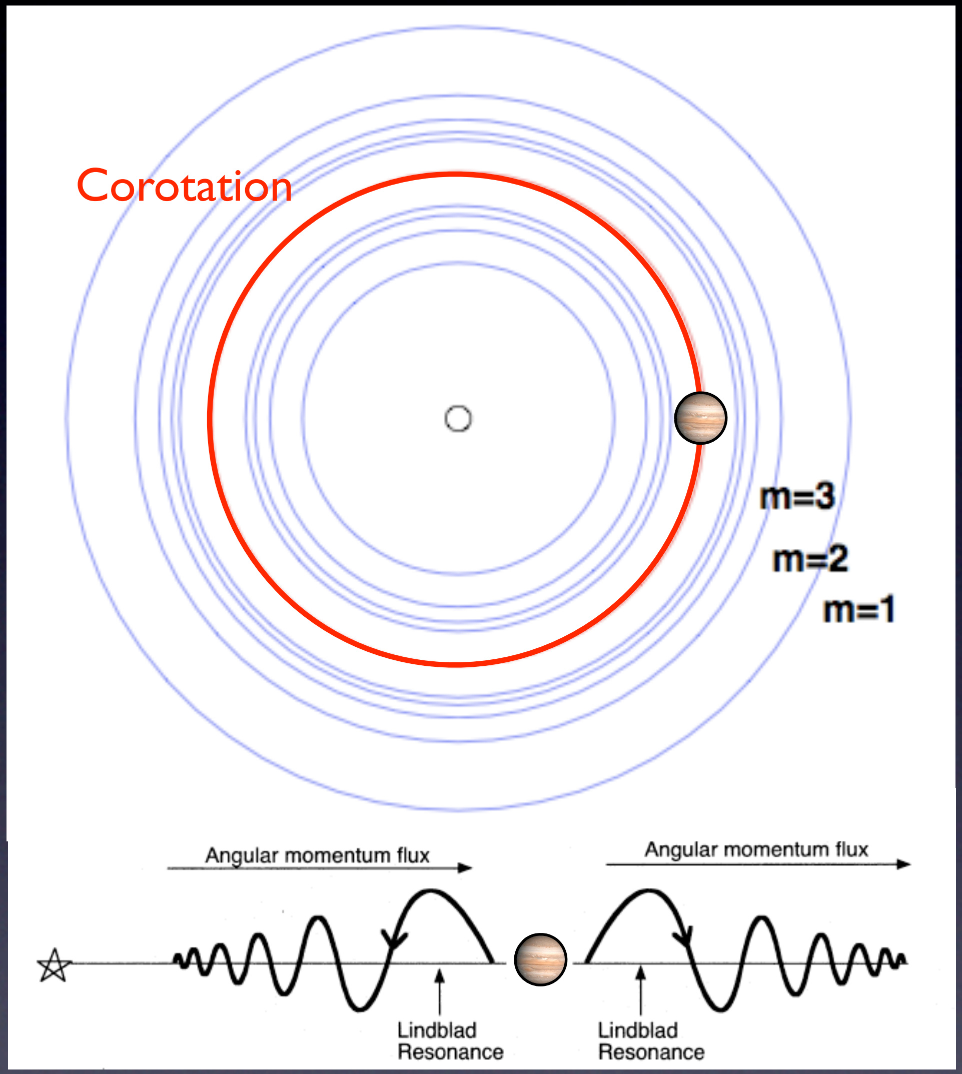


Gas disks *repel* planets



- Radial drift of meter-scale material (bottleneck)
- Torques exerted by the gas disk leads to planetary migration (Type I/II migration)

Planetary migration



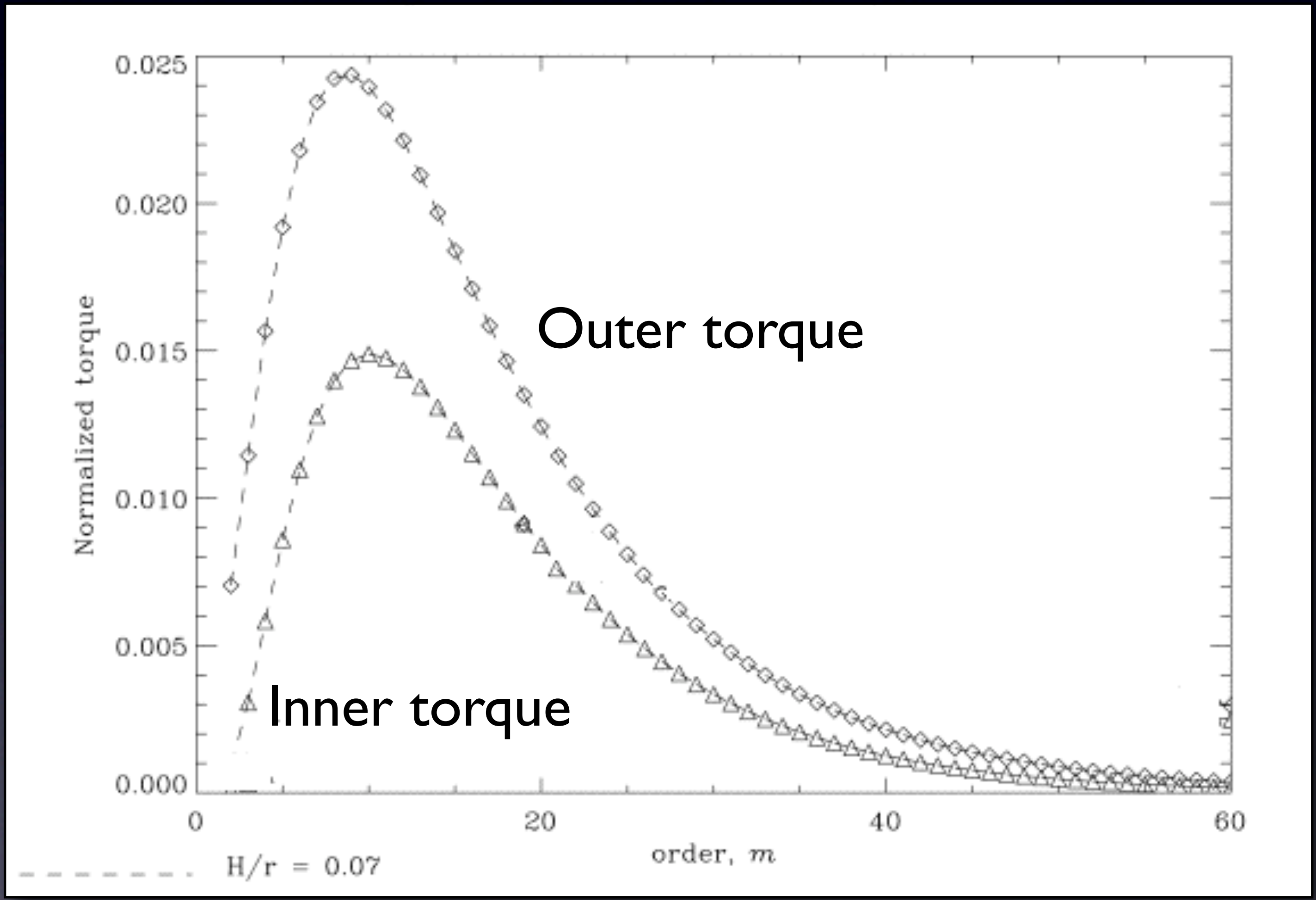
$$m(\Omega - \Omega_P) = \pm\kappa$$
$$\Omega = \Omega_P$$

- The planet transfers angular momentum to the disk at the gas resonances associated with its potential.

- Loses AM to the exterior LRs, moves closer to the star; the gas gains AM, moves outwards (conversely for ILR).

- The AM transported by the spiral wake is deposited through viscosity.

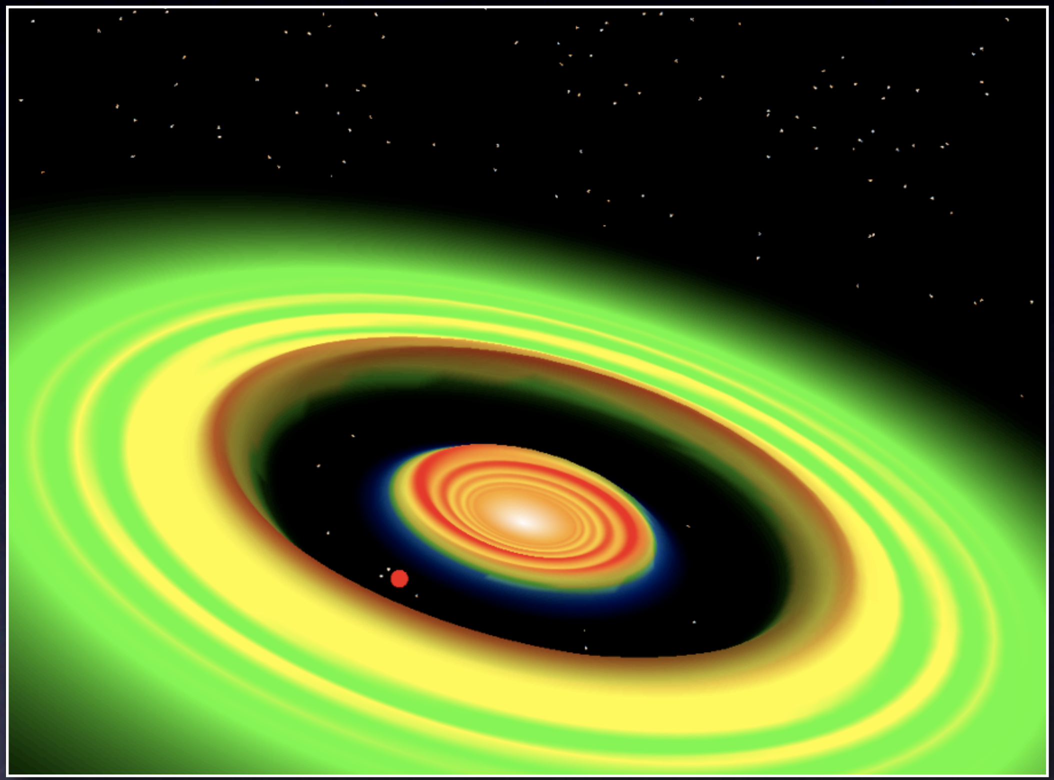
Type I migration



Masset, 2002

- The torque exerted by OLRs is generally *larger* than ILRs.
- The planet migrates *inwards* (~10^5 yrs for 5 M_{earth}).
- For low-mass planets (M ~ M_{earth}) no significant effect on surface density.

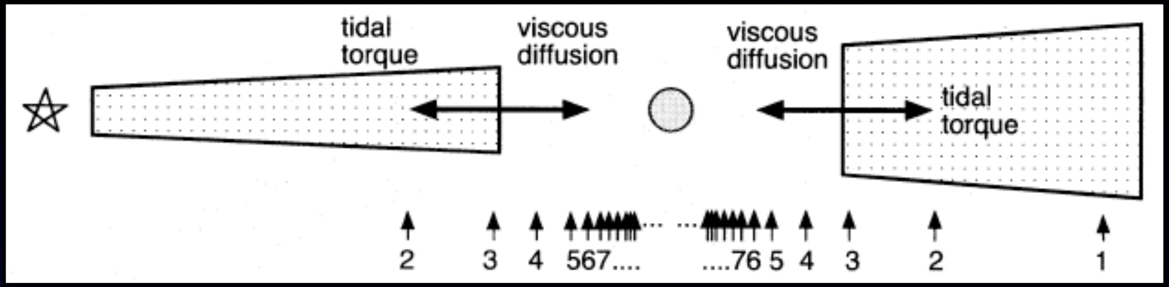
Type II migration



Bryden, 1999

- At sufficiently high masses, the AM flux dominates the viscous flux.
- A surface density depression forms around the orbit of the planet, resonances are depleted and accretion is greatly reduced.
- Orbital migration is slowed down and tied to the viscous evolution timescale.

Gap formation

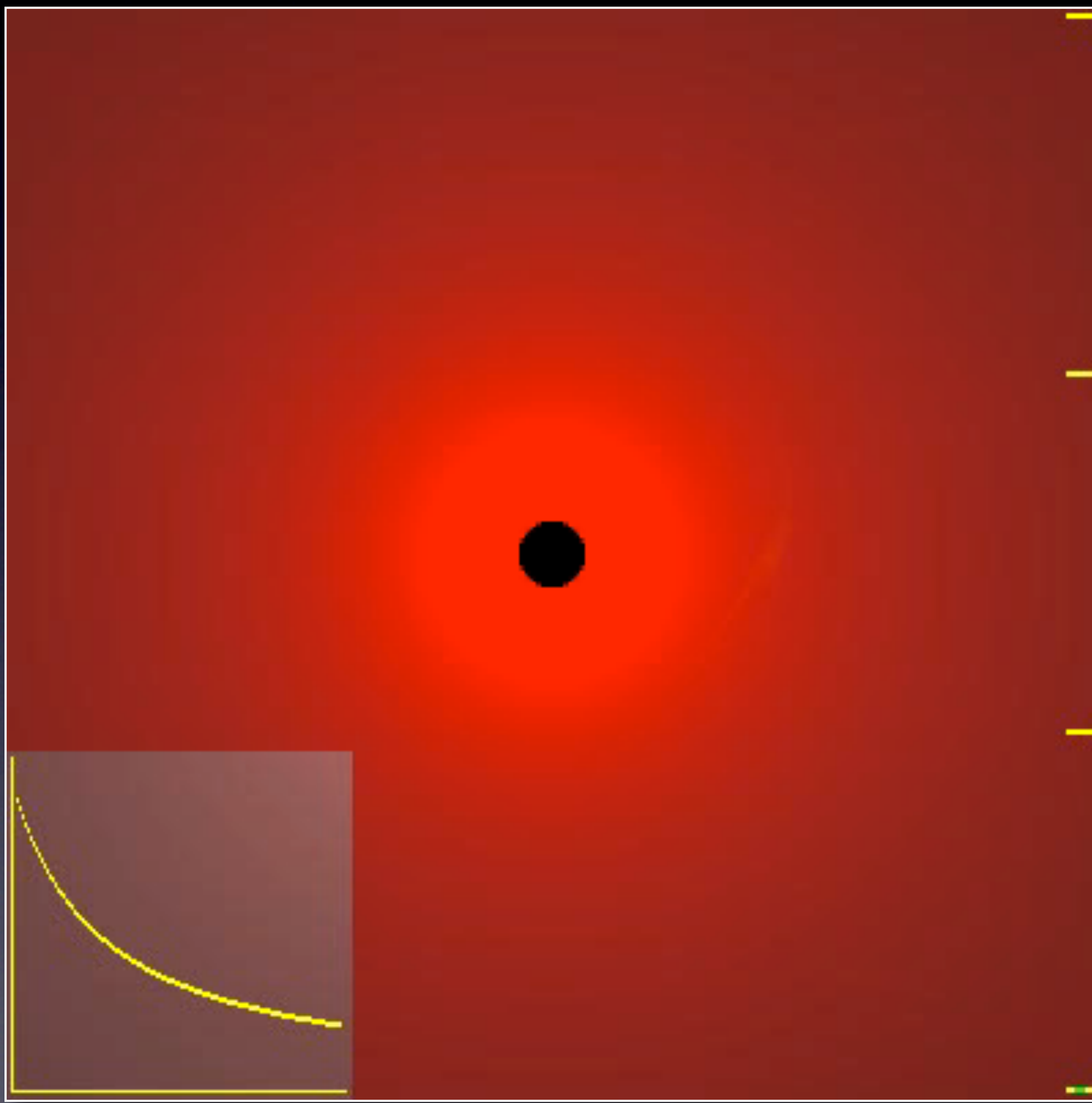


The viscous spreading timescale needs to be larger than the timescale to open a gap.

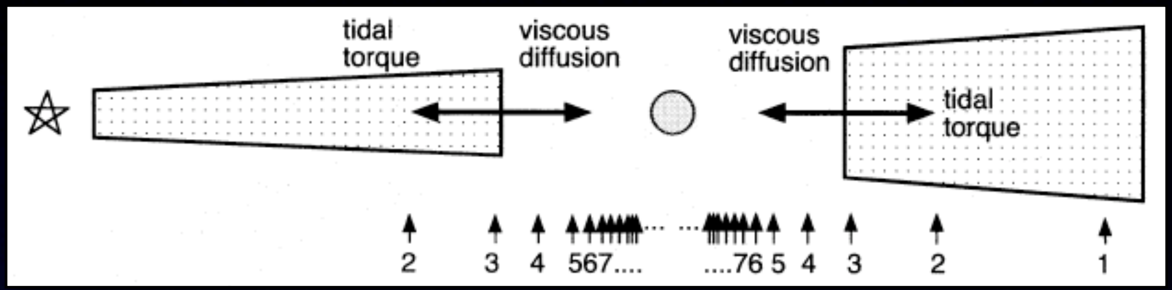
$$q > \left(\frac{c_s}{r_p\Omega_p}\right)^2 \alpha^{1/2}$$

Armitage, 2005

Gap formation



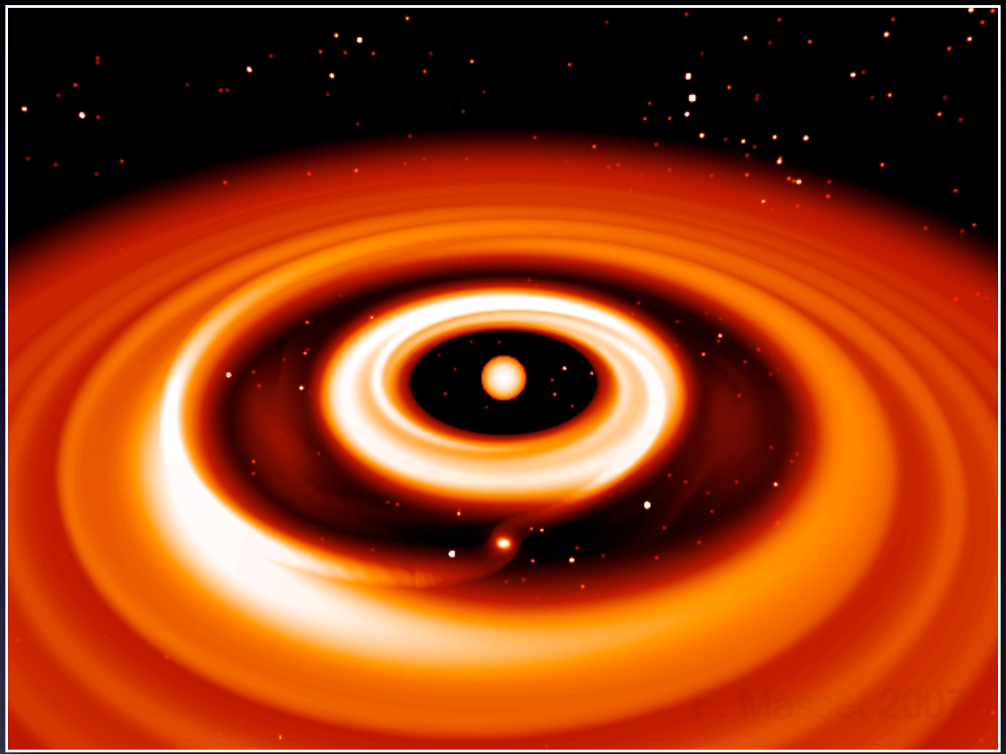
Armitage, 2005



The viscous spreading timescale needs to be larger than the timescale to open a gap.

$$q > \left(\frac{c_s}{r_p \Omega_p}\right)^2 \alpha^{1/2}$$

Gap opening criteria



Masset, 2007

- Gap opening bridges Type I migration to the slower Type II migration.
- It determines the accretion rate onto the giant planet.

This is crucial to our understanding of giant planet formation timescales! Are we accounting for everything?

Role of self-gravity

$$Q = \frac{\kappa c_s}{\pi G \sigma}$$

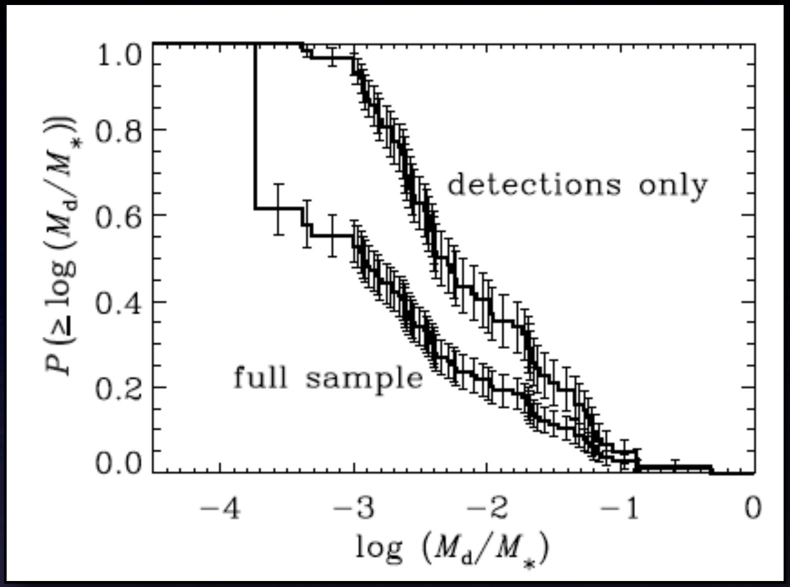
- *Laughlin & Bodenheimer, 1994* found that disks with appropriate ranges of Q might be unstable to nonaxisymmetric disturbances, leading to mass + angular momentum transport and heating of the disk.

- *Nelson & Benz, 2003* found significant discrepancies in the minimum gap-opening mass and migration rate with respect to the analytical formulas, and depending on self-gravity.

- *Baruteau & Masset, 2008* show that self-gravity significantly accelerates type I migration.

But self-gravity is often not included to reduce computational complexity...

Role of self-gravity



Andrews & Williams, 2005

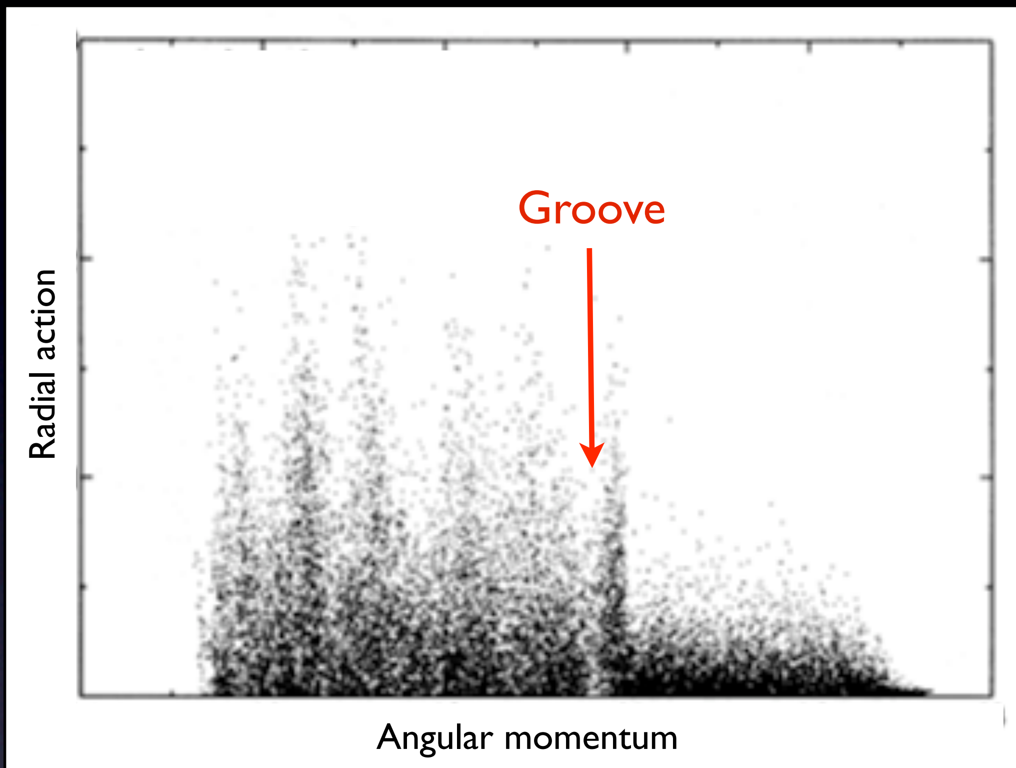
- Observed disks are generally not very massive (median mass ~ 0.5% M*, i.e. Andrews & Williams 2005); MMSN ~ 0.03 M*.

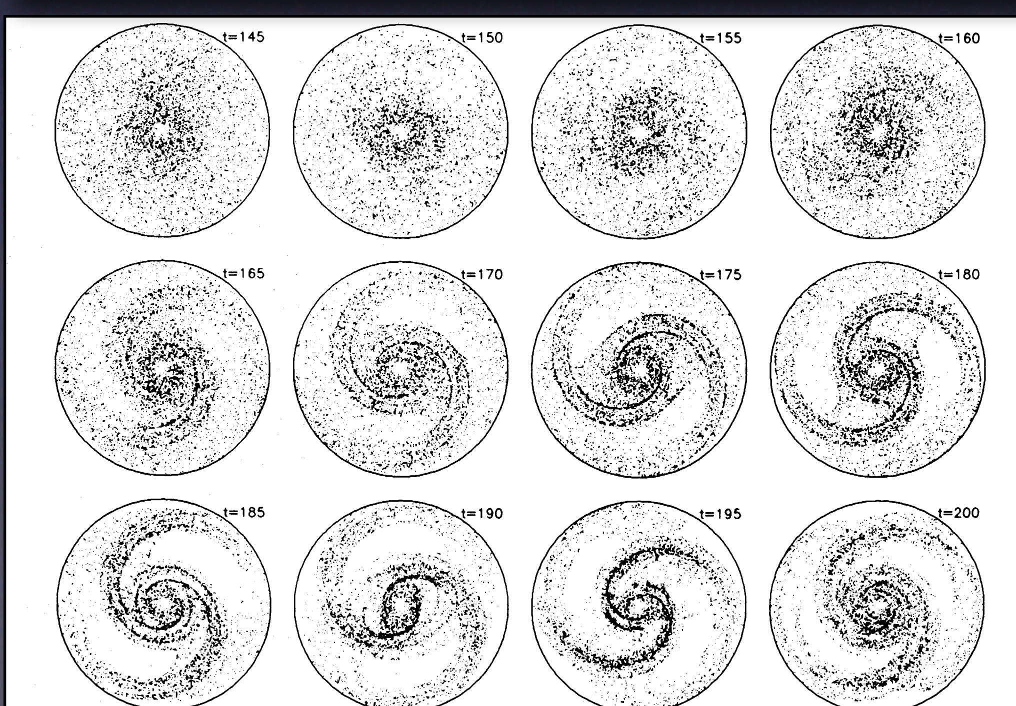
- But gravitational instability commonly requires

$$Q = \frac{c_s \kappa}{\pi G \sigma} < 1 - 1.5 \quad [M_D/M_\odot > 0.1 - 0.2]$$

Can we still excite Gravitational Instability in moderate to low mass disks? Maybe...

Groove modes

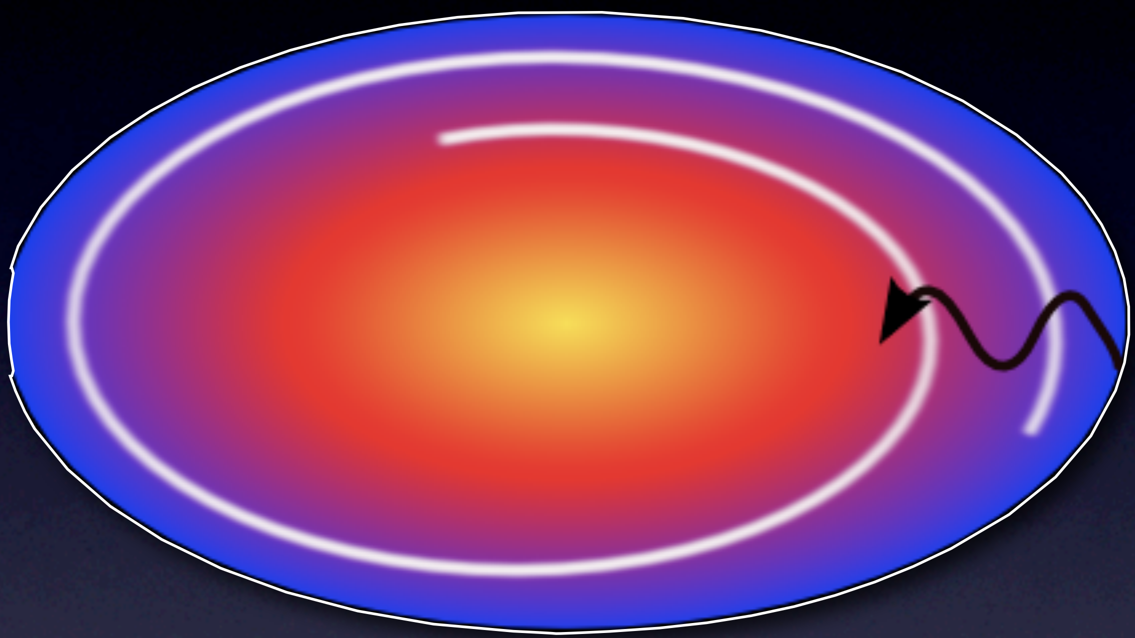


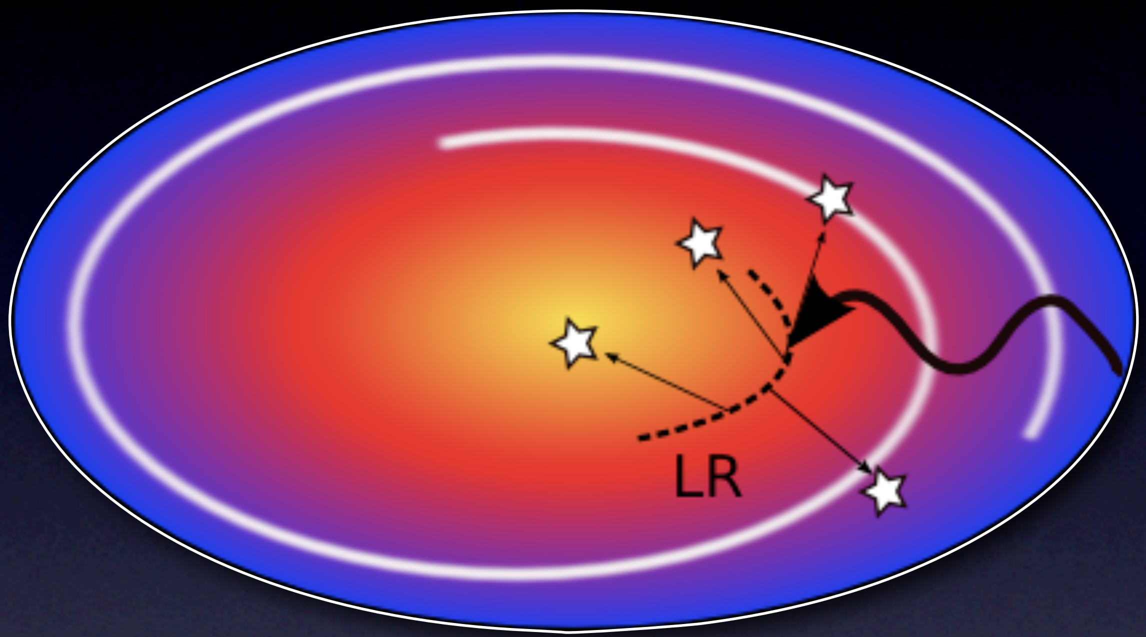


- Sellwood & Lin, 1989 discovered a new class of spiral instabilities in stellar disks they called "*groove modes*".

- This violent instability is triggered by a *groove* in the phase space distribution of particles, which corresponds to an annular density depression in physical space.

Feedback cycle in ★ disks





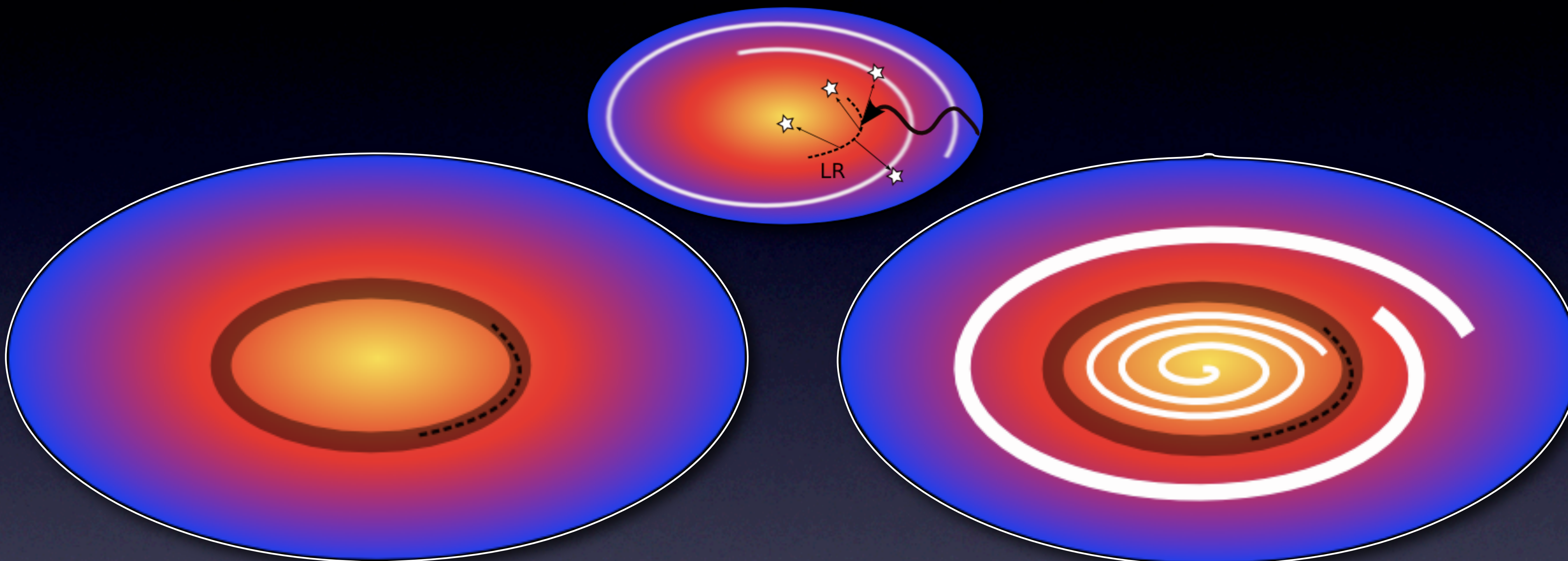
1. Slow mode

A slow-growing spiral packet (i.e. an edge mode) becomes unstable and propagates in the disk with a group velocity;

2. LR particle scattering

The packet reaches one of its LRs and scatters particles from the narrow resonance;

Feedback cycle in ★ disks



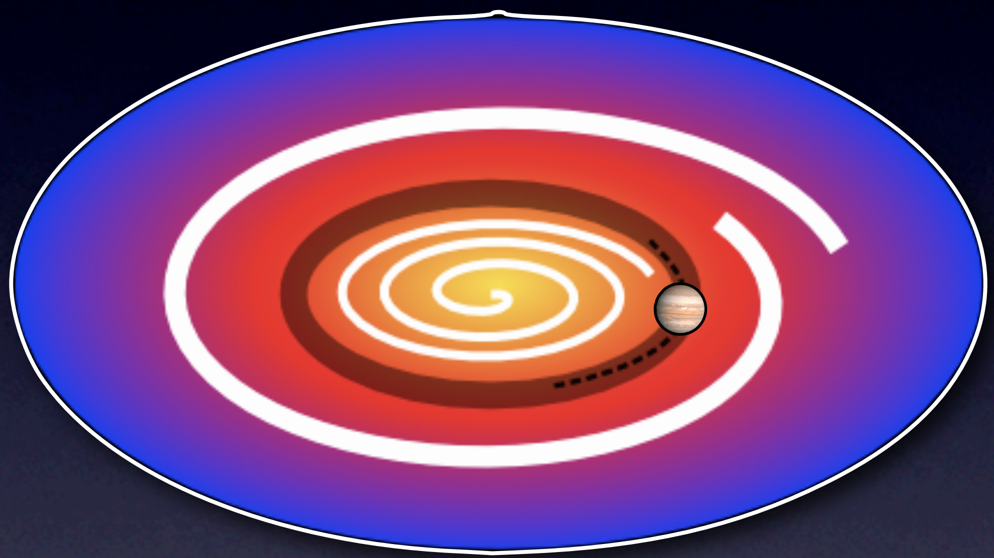
3. Density groove

The wave-particle interaction creates a groove in surface density;

4. Groove mode

A fast-growing groove mode is launched (and can carve new grooves) → *feedback cycle*

Groove modes in gas disk?



- The feedback cycle cannot be supported in gaseous disks because it relies on the *wave-particle* interaction at resonances.
- *<u>But</u>* a gap is naturally carved (and mantained) during giant planet formation!

GI might depend not only on disk mass and sound speed, but also on the density profile...

Can we excite GIs in a low mass disks with a gap?

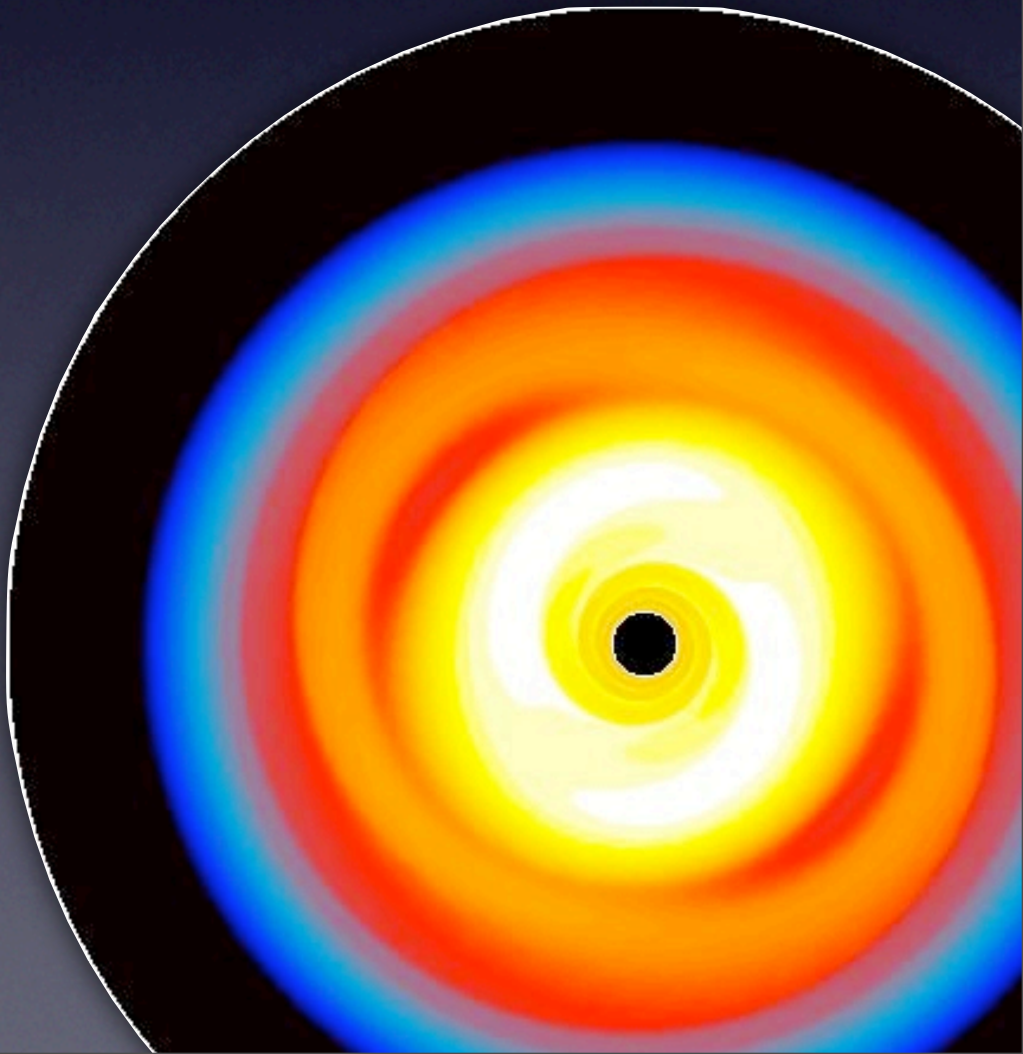
Program

If GIs are excited by grooves even in low-mass disks, then they could significantly influence disk evolution.

- Demonstrate convincingly the emergence of groove modes in gas disks with gaps, at masses comparable to observations.

- Compare to disks without gaps.

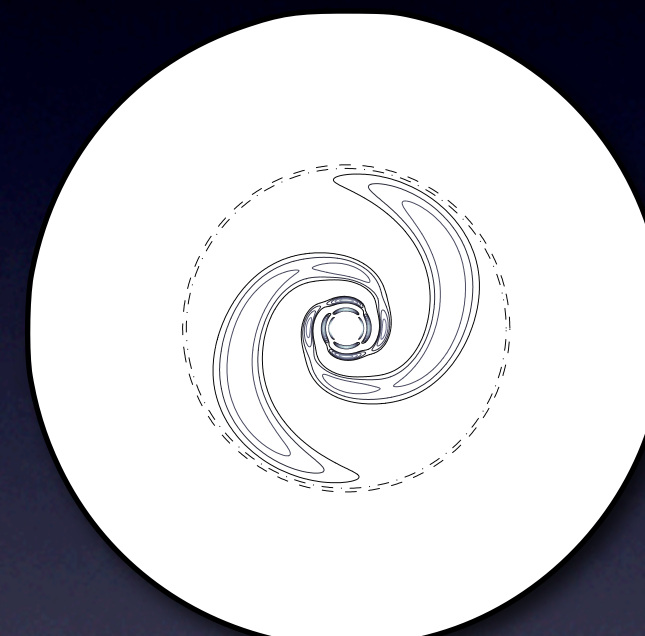
- Study the effect of the GI on the disk and the gap.



Methods

- Modal analysis

  Predict growth rate and pattern speed of an intrinsic mode of the disk by solving a generalized eigenvalue problem, in the limit of linear perturbations. Fast and accurate in the linear regime.
  *Pannatoni & Lau, 1979; Adams, Ruden and Shu (**ARS**), 1989*



- Nonlinear simulations

  Do a full hydrodynamical simulation, measure properties of the emerging mode *a posteriori.*

Modal analysis

Assume a linear perturbation of the form

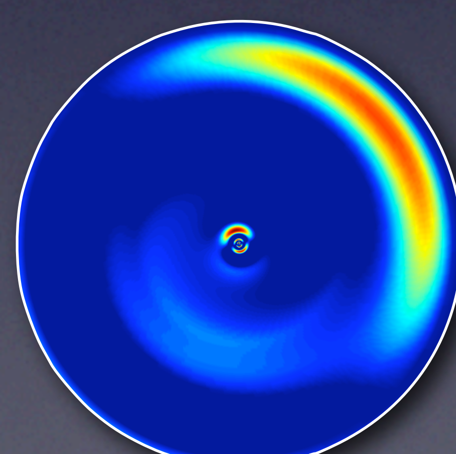
$$F(r, \varphi, t) = F_1(r) \exp\left[i(\omega t - m\varphi)\right] \qquad [F = \Sigma, u, v, \Psi]$$

$$\omega = m\Omega_P - i\gamma$$

$m\Omega_P$ → pattern speed

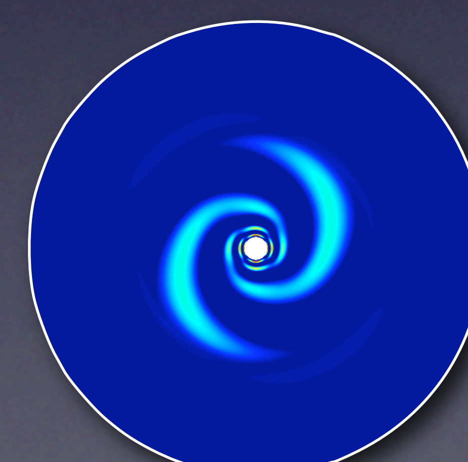
$i\gamma$ → growth rate

$m = 0$



$m = 1$

$m = 2$



$m = 2$

Modal analysis

Solve the equations of hydrodynamics to linear order

$$
\begin{aligned}
i(\omega - m\Omega)\frac{\sigma_1}{\sigma_0} + \left(\frac{1}{r} + \frac{1}{\sigma_0}\frac{d\sigma_0}{dr}\right)u_1 + \frac{du_1}{dr} - \frac{im}{r}v_1 &= 0 \\
i(\omega - m\Omega)u_1 - 2\Omega v_1 &= -\frac{d}{dr}(\Psi_1 + h_1) \\
i(\omega - m\Omega)v_1 + \frac{\kappa^2}{2\Omega}u_1 &= \frac{im}{r}(\Psi_1 + h_1) \\
\Psi_1 &= -2\pi G \int K_m(r,\rho)\sigma_1(\rho)d\rho
\end{aligned}
$$

- Assume spiral arms to be “tightly wrapped” (WKB analysis), <u>or</u>

- *Solve as a matrix equation on a radial grid, get global modes (ARS)*

Modal analysis

Combine the set of equations of hydrodynamics in terms of operators [Lau & Bertin 1978]:

$$L\left[a_0^2\frac{\sigma_1}{\sigma_0}+\Psi_1\right]+Ca_0^2\frac{\sigma_1}{\sigma_0}=0$$

- Rewrite differential and integral operators on a radial grid.
- Getting an accurate estimation of the potential is surprisingly hard!

$$\Psi(R)=-\int_0^\infty \sigma(\rho)\rho d\rho\int_0^{2\pi}\frac{d\varphi}{\sqrt{R^2+\rho^2-2R\rho\cos\varphi}}$$

- Get a generalized eigenvalue matrix equation:

$$Z_{ik}(\omega)S_k=0$$

[solution if ω makes the determinant = 0]

Modal analysis

$$Z_{ik}(\omega_R, \omega_I) S_k = 0$$

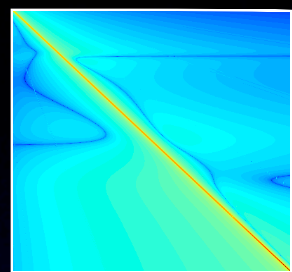
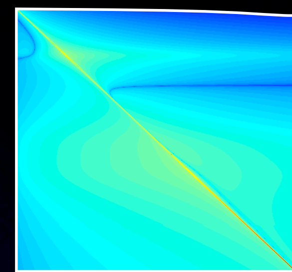
- Solve the eigenvalue problem for a given *m*, get a variety of solutions for ω.

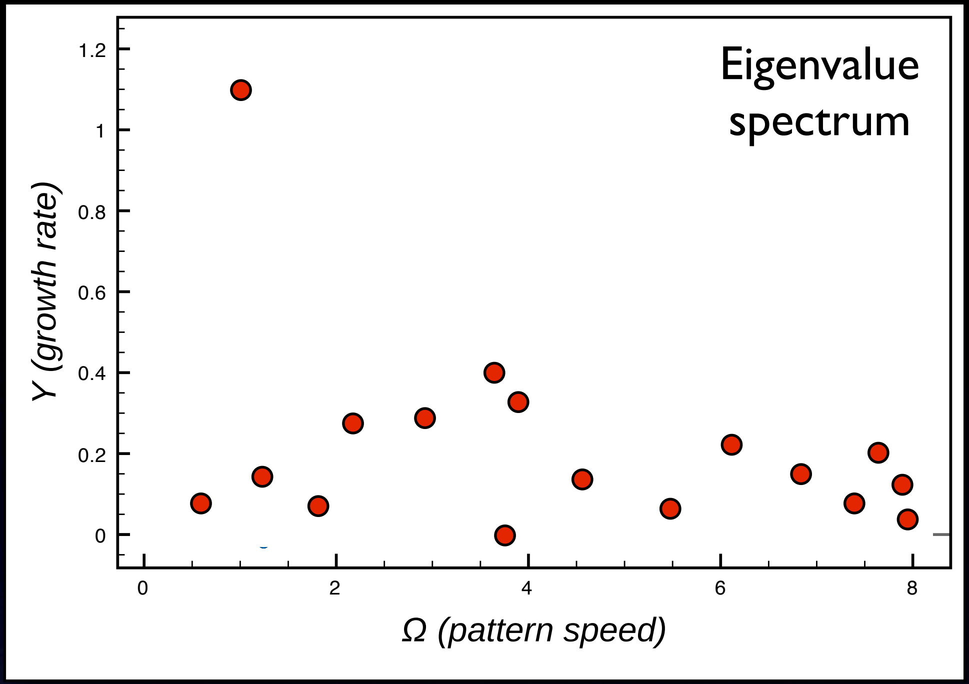
ϖ_R ⟶ pattern speed

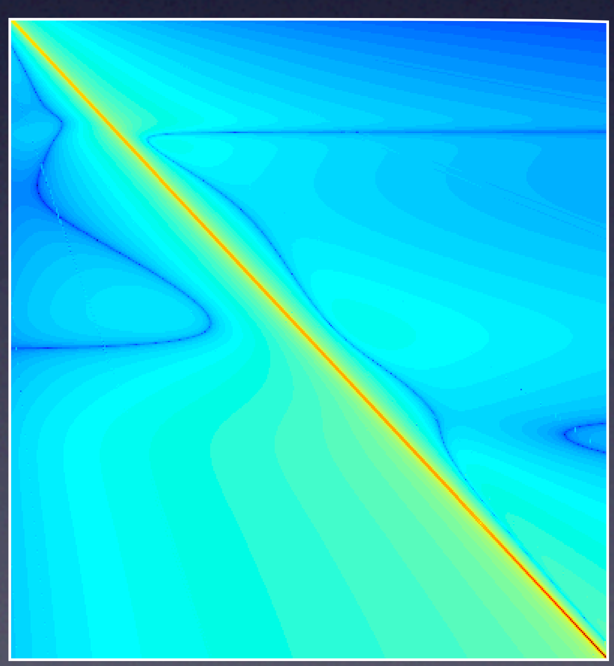
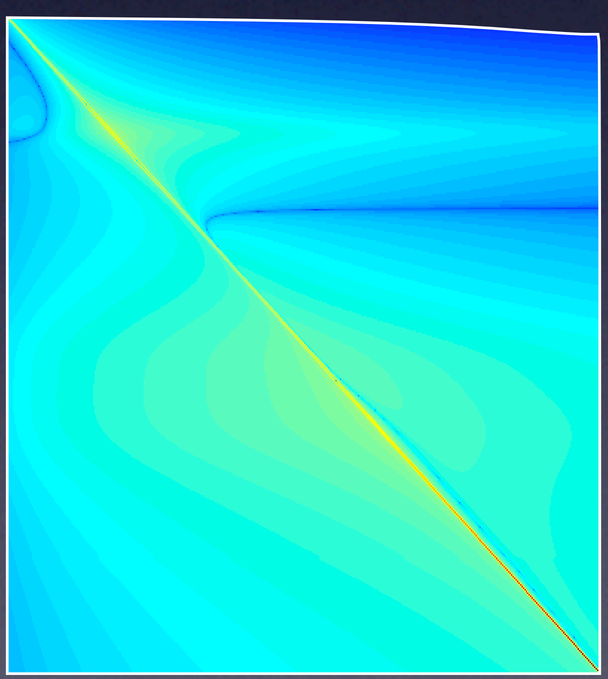
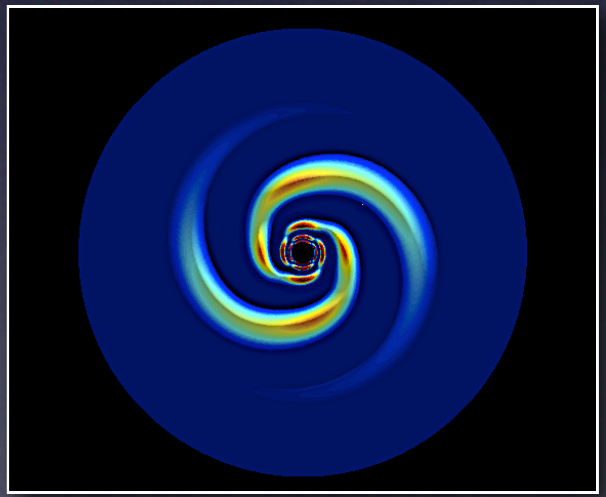
ϖ_I ⟶ growth rate

$S_{k=1..400}$ ⟶ radial profile

This calculation is not trivial, because need to find the roots of the determinant of the real and imaginary parts of a 400x400 [800x800] matrix simultaneously and with high accuracy.

det [ + *i* ] (ω) = 0



[ + *i* ] ×  = 0

Z_{ik}

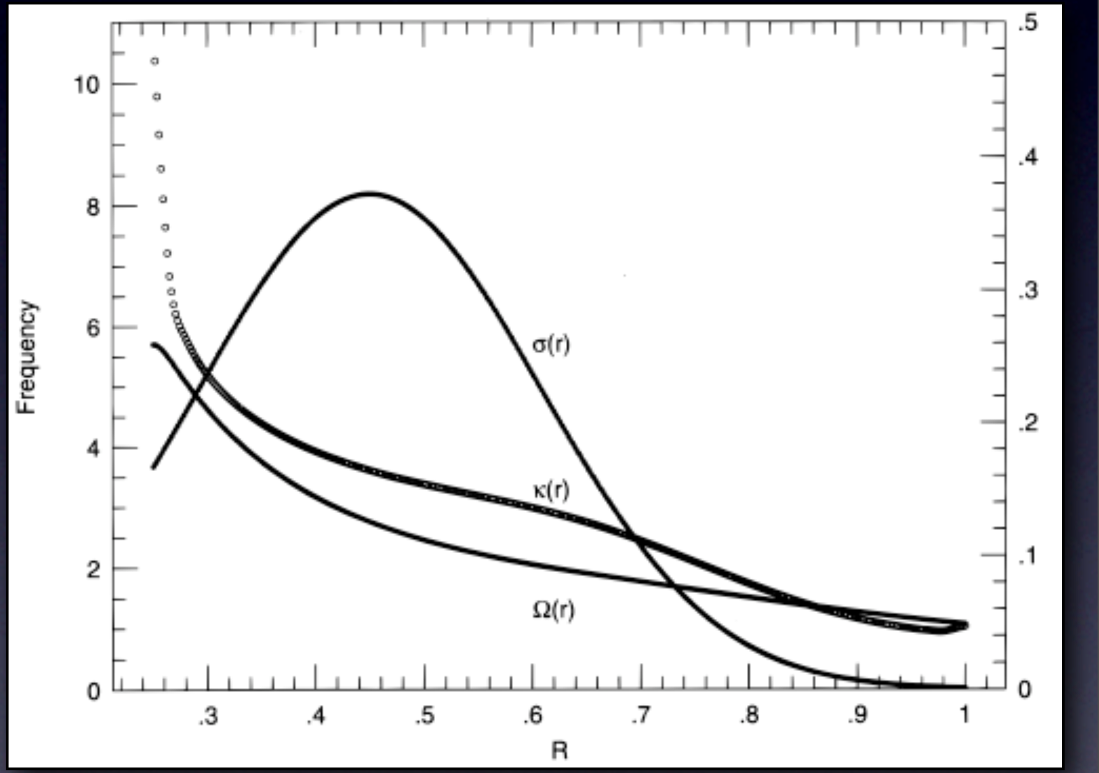
S_k

eigenfunction

"Base" equilibrium model

We consider a Gaussian density profile with a polytropic EOS:

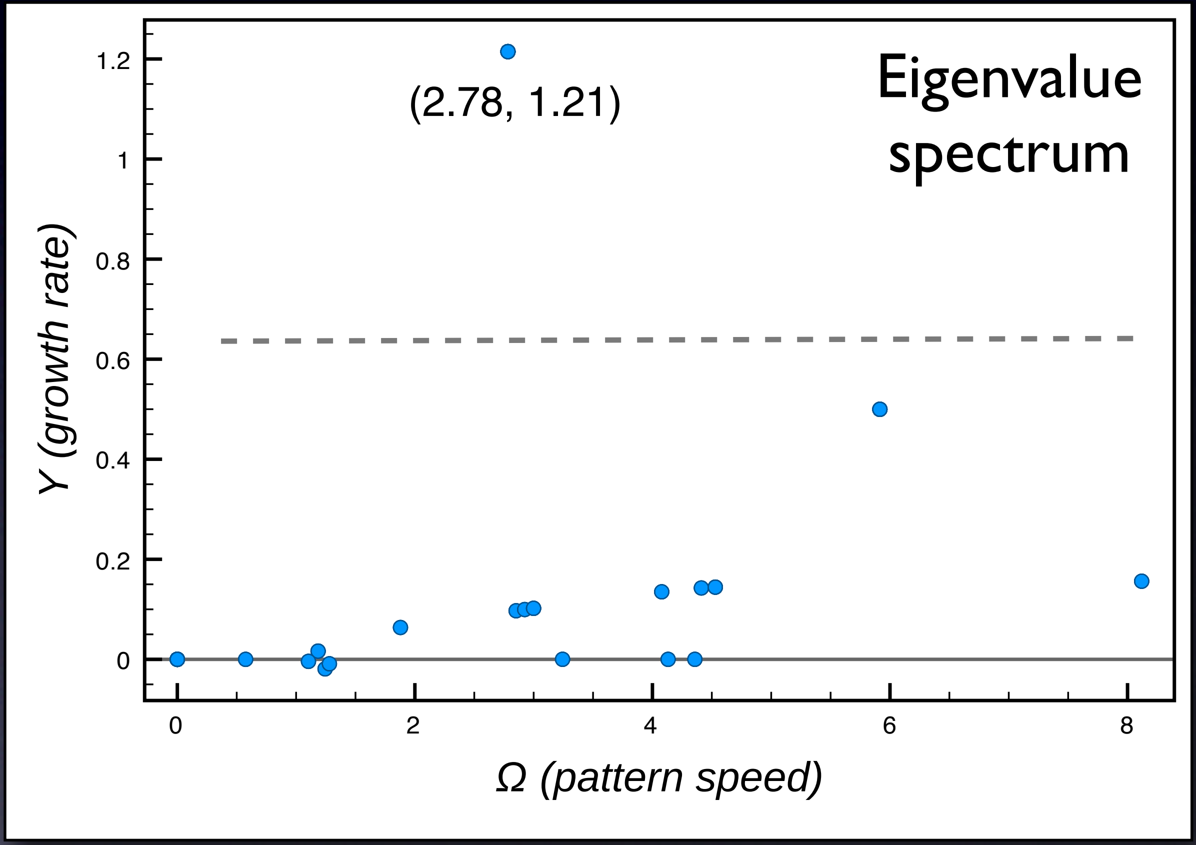
$$\Sigma(r) = \Sigma_0 \exp\left(-(r-R_0)^2/w\right)$$
$$c_s^2 = K\gamma\sigma^{\gamma-1}$$



This is the "reference disk" considered by Laughlin & Rozyczka (1996) and the model we concentrate on.

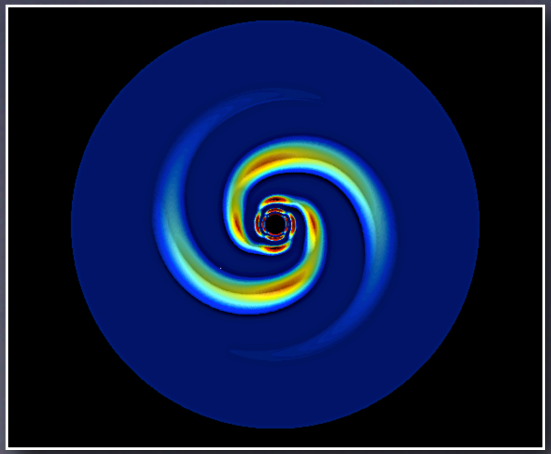
“Base” equilibrium model

Set the normalizations to yield $q_D = 1$, $Q_{min} = 1.22$.



No pretense of being a particularly faithful model for protoplanetary disks.

But it is simple and has the advantage of possessing a single $m = 2$ mode clearly identifiable both in the linear and nonlinear simulations.



Eigenfunction

Nonlinear simulations

- We complement and verify the results of the modal analysis by running a hydrodynamical simulation.
- Start with a disk in rotational equilibrium and disturb it with a small random perturbation.
- The hydrodynamical simulation can follow the time evolution of a spiral mode from the linear growth regime all the way to saturation.
- Can measure approximate growth rate and pattern speed of the emerging mode.

Nonlinear simulations

- Solve the equations of continuity and momentum of a thin, polytropic, self-gravitating disk.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \phi} - \frac{v^2}{r} = -\frac{\partial h}{\partial r} - \frac{\partial \Psi}{\partial r} - \frac{\partial \Psi_*}{\partial r},$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \phi} + \frac{vu}{r} = -\frac{1}{r}\frac{\partial h}{\partial \phi} - \frac{1}{r}\frac{\partial \Psi}{\partial \phi} - \frac{1}{r}\frac{\partial \Psi_*}{\partial \phi},$$

$$\frac{\partial \sigma}{\partial t} + \frac{1}{r}\frac{\partial r\sigma u}{\partial r} + \frac{1}{r}\frac{\partial \sigma v}{\partial \phi} = 0 .$$

- Solved on a 256x256, 2-D polar grid; reflective boundary conditions.

- Two-dimensional Euler differencing scheme, with second-order van Leer-type advection.

Nonlinear simulations

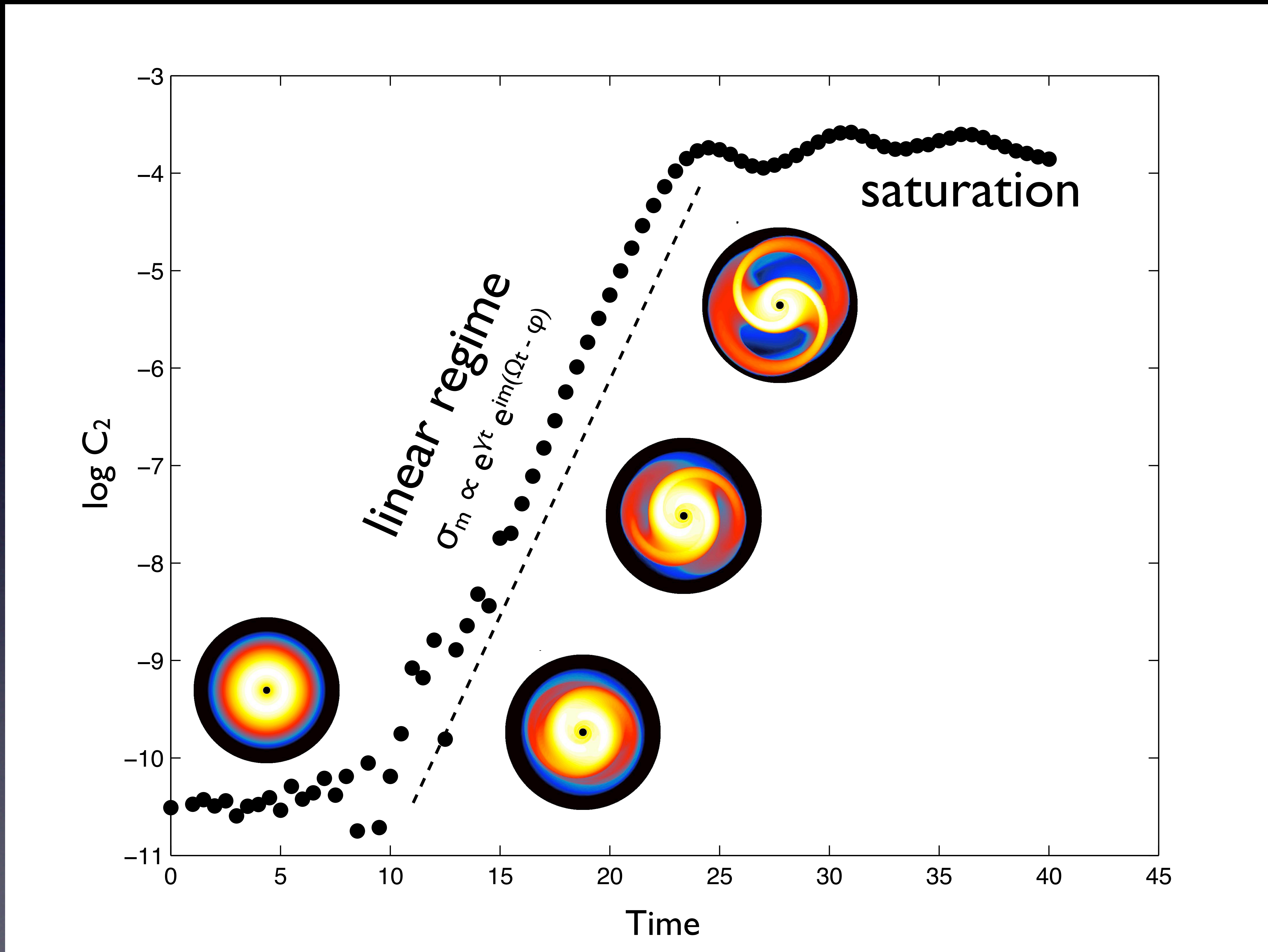
- The gravitational potential of the disk is calculated by applying the 2-D Fourier convolution theorem.

- Quantify the growth of a spiral mode by looking at the Fourier decomposition of the surface density

$$C_m = \frac{\int \int_0^{2\pi} \sigma(r,\varphi) e^{-im\varphi} d\varphi dr}{\int \int_0^{2\pi} \sigma(r,\varphi) d\varphi dr}$$

$$\gamma_m = \frac{d}{dt} \log C_m \longrightarrow$$

direct comparison with mode analysis


log C_2
Time
linear regime
$\sigma_m \propto e^{\gamma t} e^{im(\Omega t - \varphi)}$
saturation
−3
−4
−5
−6
−7
−8
−9
−10
−11
0
5
10
15
20
25
30
35
40
45

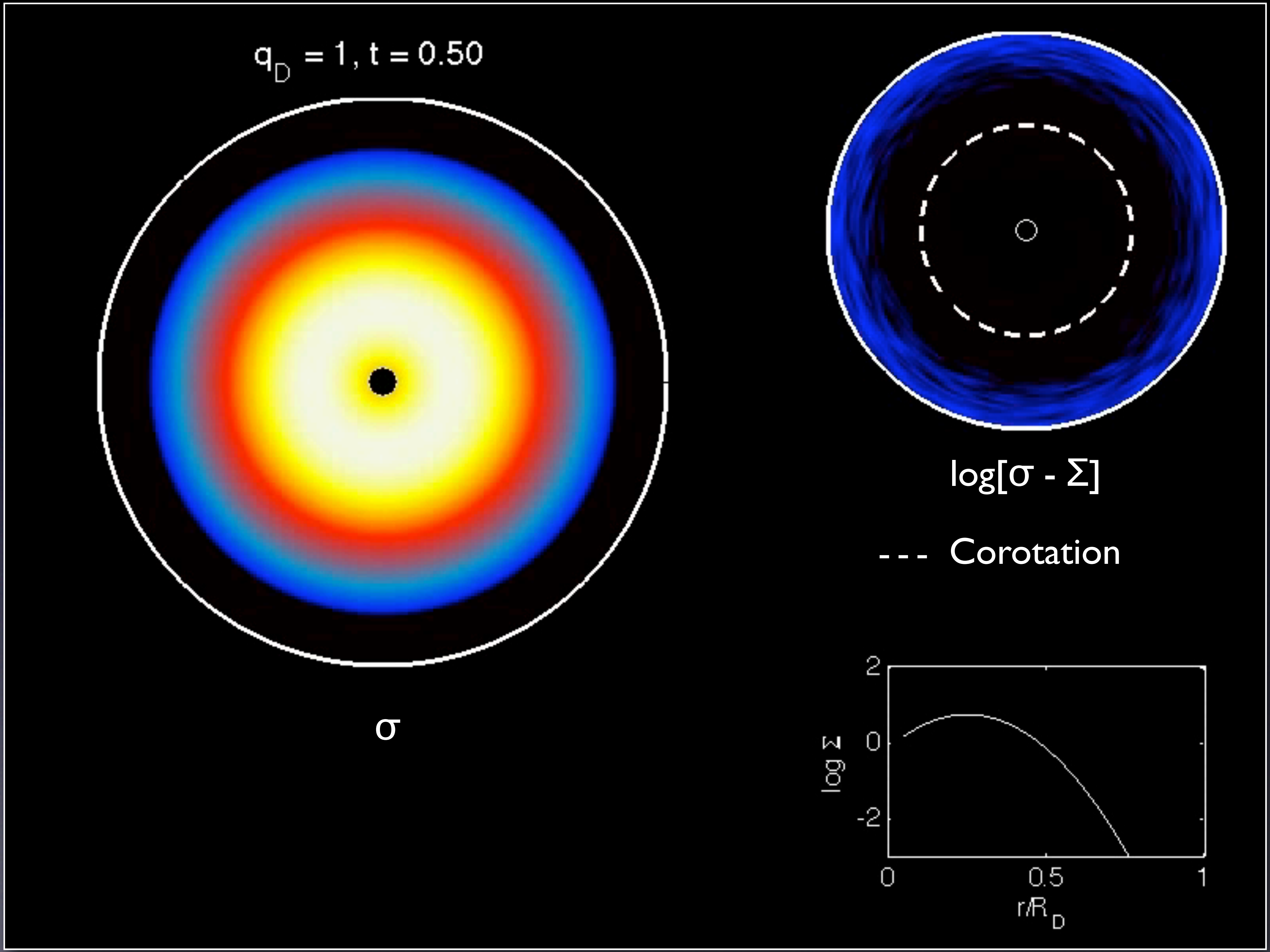
Base disk, $q_D = 1$

log[σ - Σ]

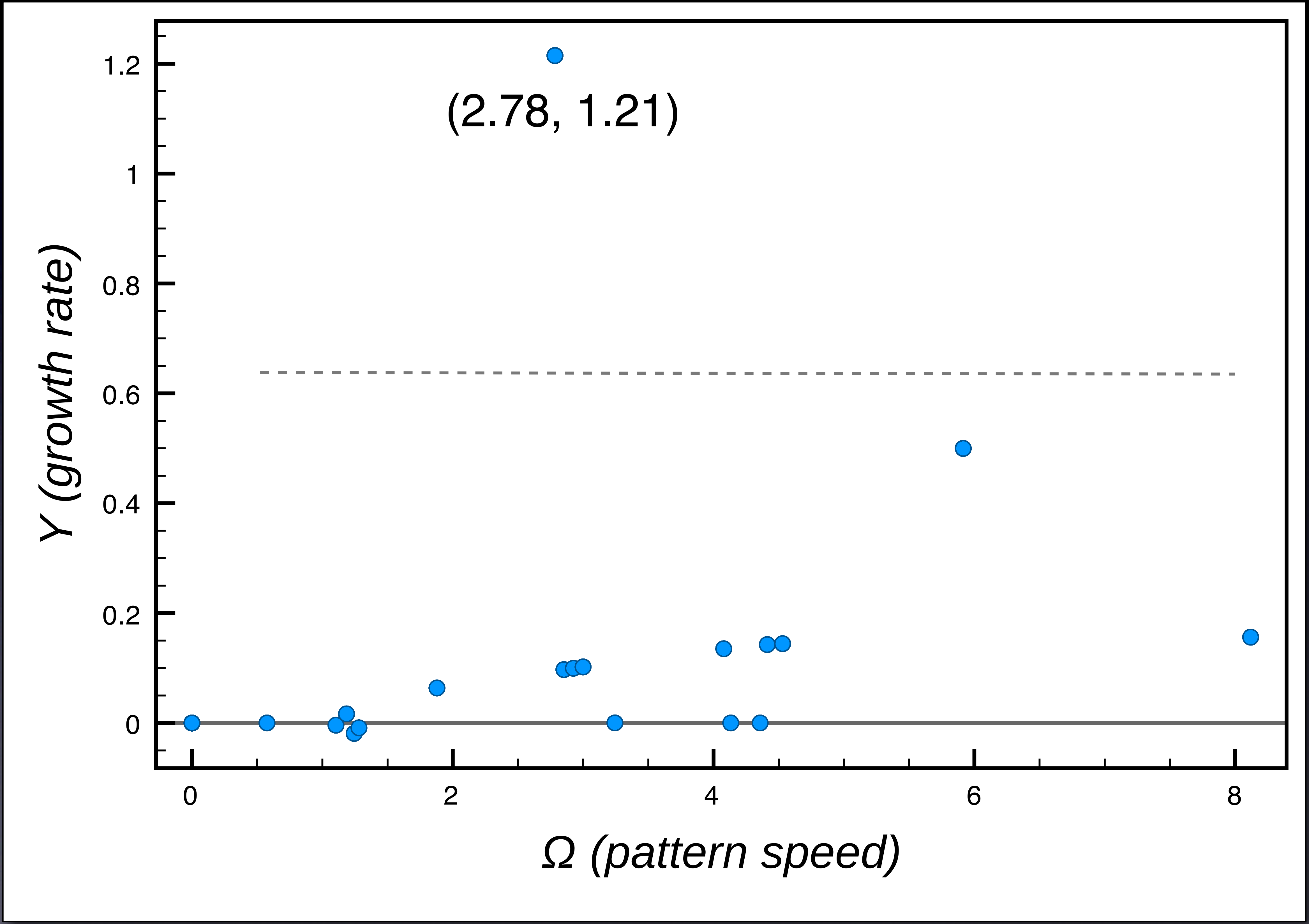
- - - Corotation

σ

Base disk, $q_D = 1$



Eigenvalue spectrum



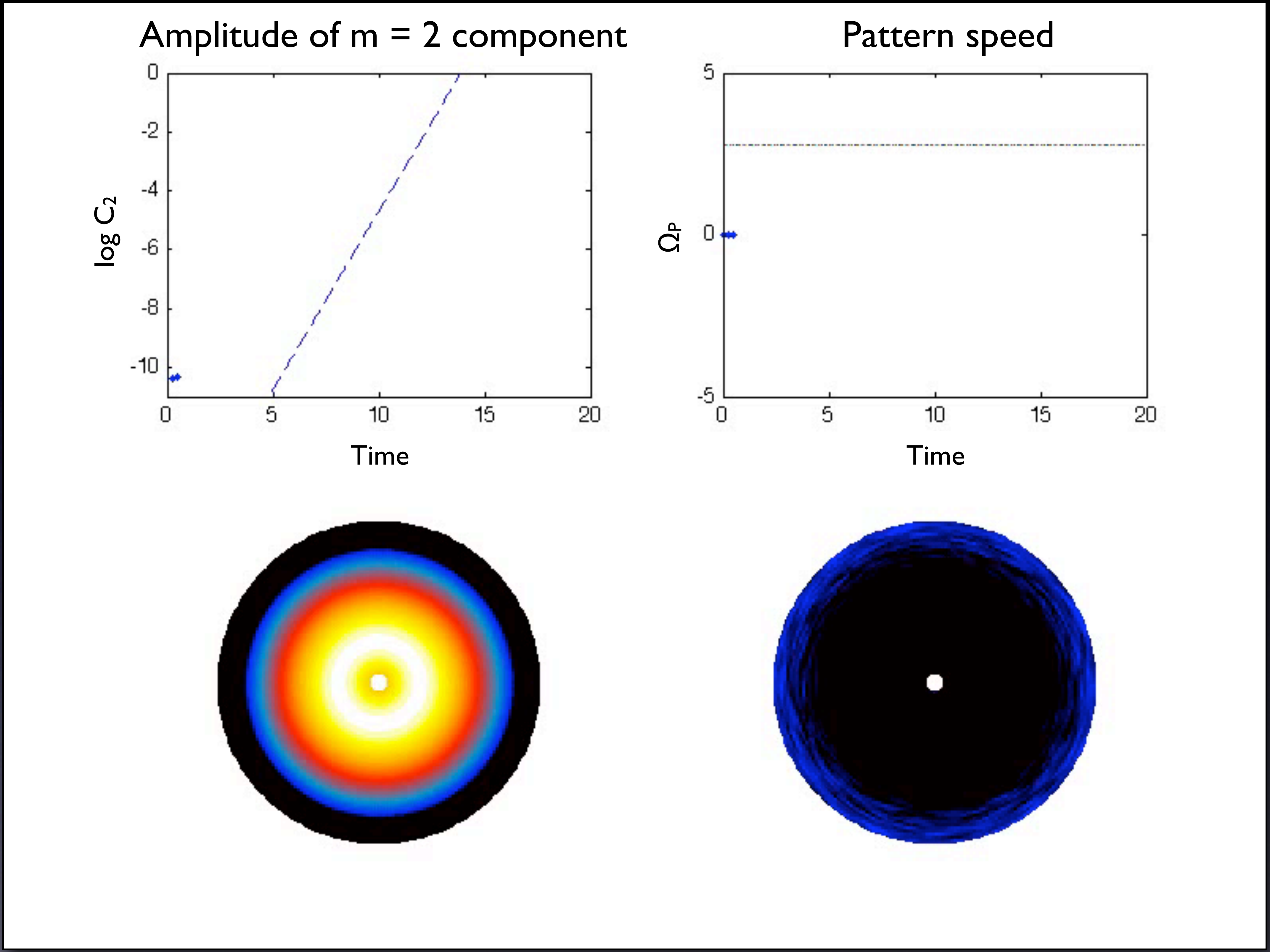
Base disk

$\log C_2$

Ω_P

Time

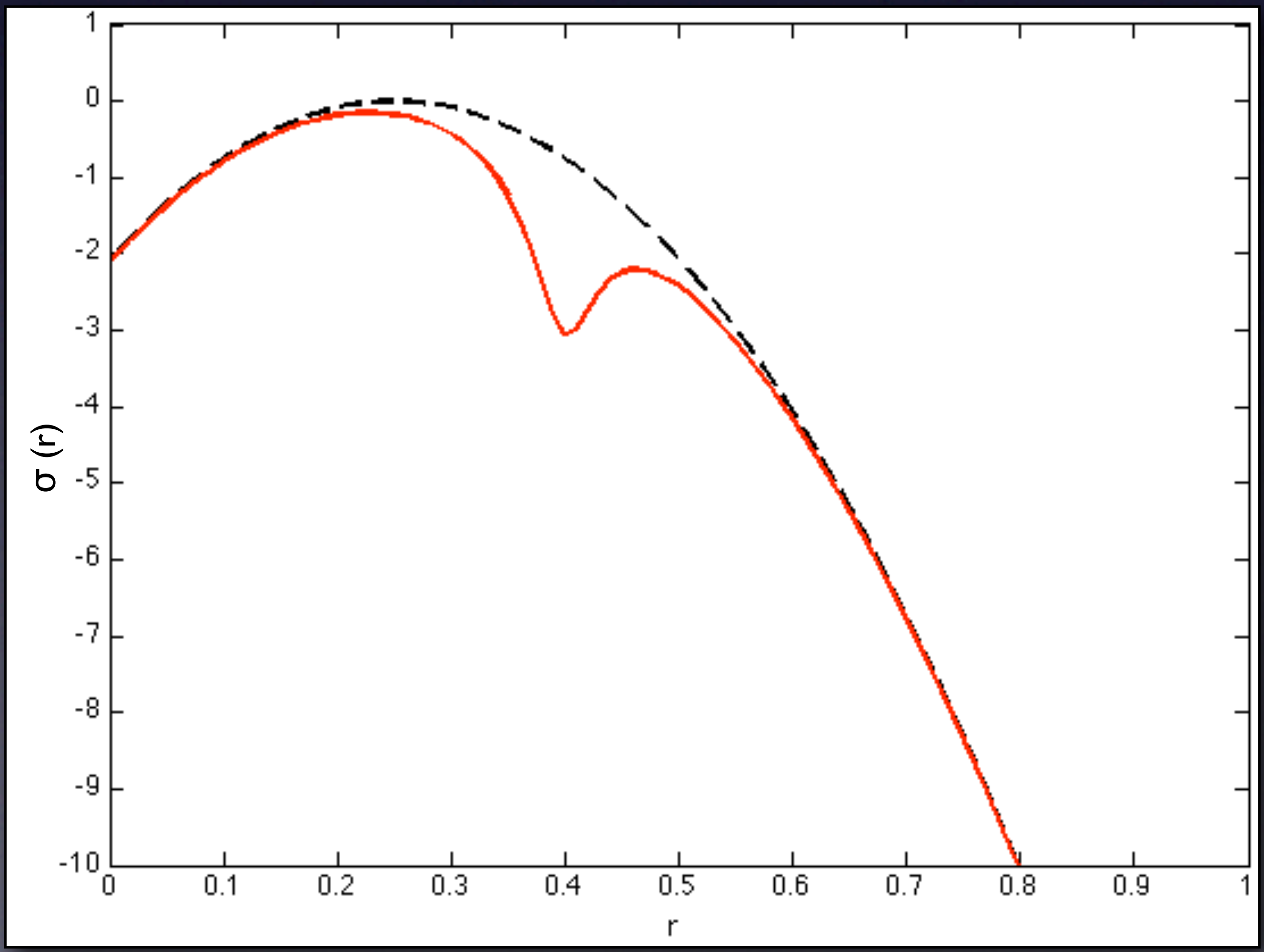
Time


Amplitude of m = 2 component
Pattern speed
log C2
ΩP
Time
Time
0
-2
-4
-6
-8
-10
5
0
-5
0
5
10
15
20

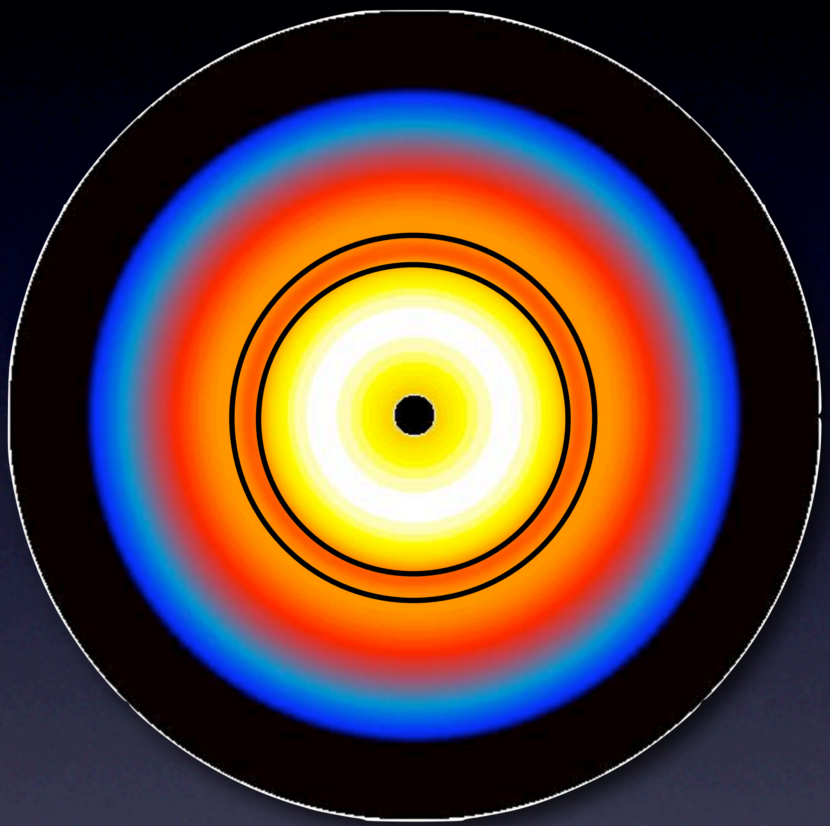
Models with gap

$$\sigma(r) = \sigma_0 e^{\left[-(r-R_0)^2/w\right]} \times \left(1 - \frac{A\Delta^2}{(r-R_P)^2 + \Delta^2}\right)$$

Groove with depth *A* centered around R_P and characteristic width Δ



Models

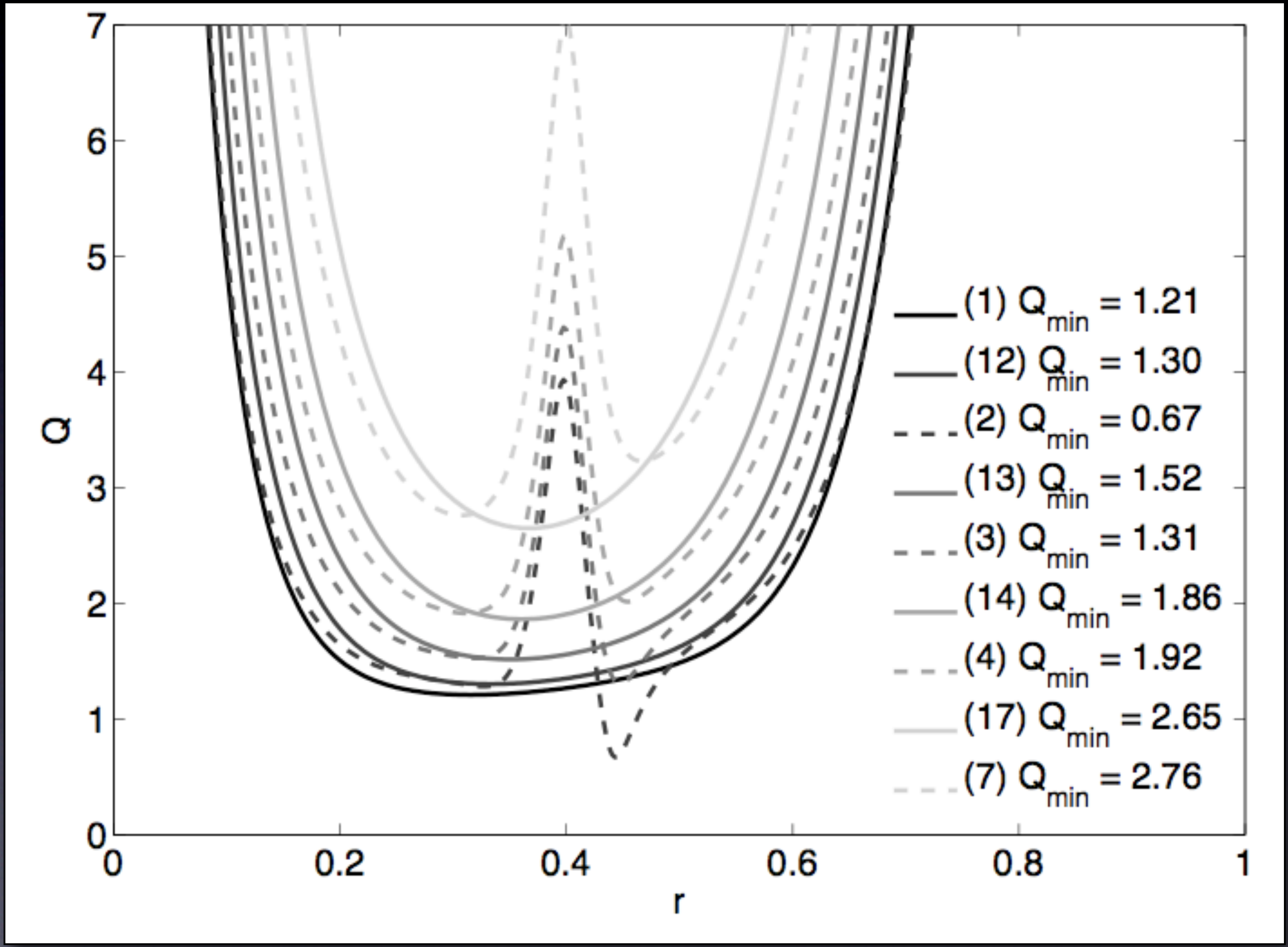


We do not include the potential of the planet and carve a gap “ab initio.”

This singles out modes that are intrinsic to disk+groove vs. spiral wakes driven by the planet.

Models

Model	q_D	A	Q_{min}	m
1	1	0	1.21	2
2	0.63	0.90	0.67	2
3	0.32	0.90	1.31	2
4	0.16	0.90	1.92	2
5	0.13	0.90	2.03	2
6	0.08	0.90	2.24	2
7	0.06	0.90	2.76	2
12	0.63	0	1.30	2
13	0.32	0	1.52	2
14	0.16	0	1.86	2
15	0.13	0	2.08	2
16	0.08	0	2.26	2
17	0.06	0	2.65	2



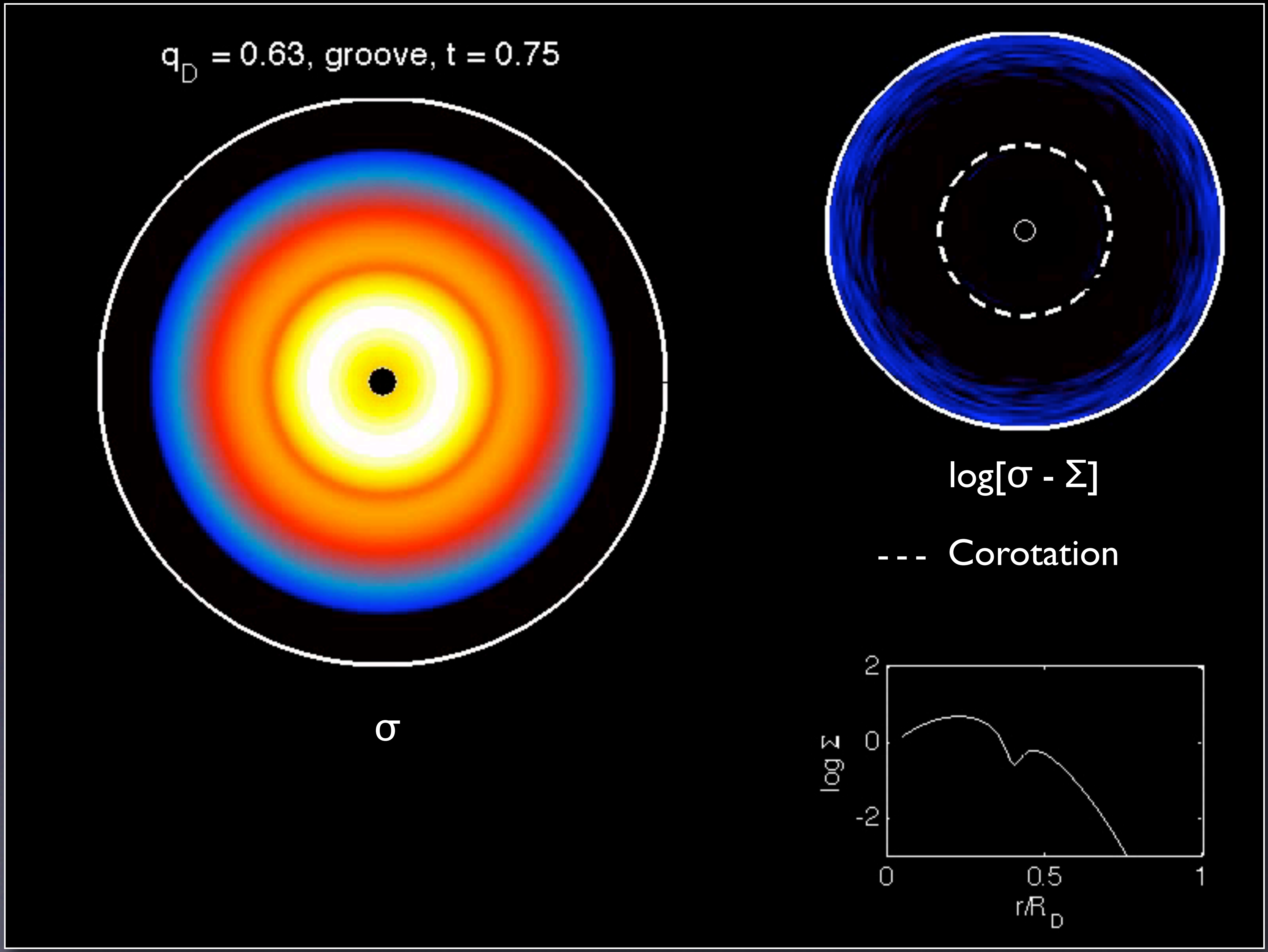
Disk with groove, $q_D = 0.63$

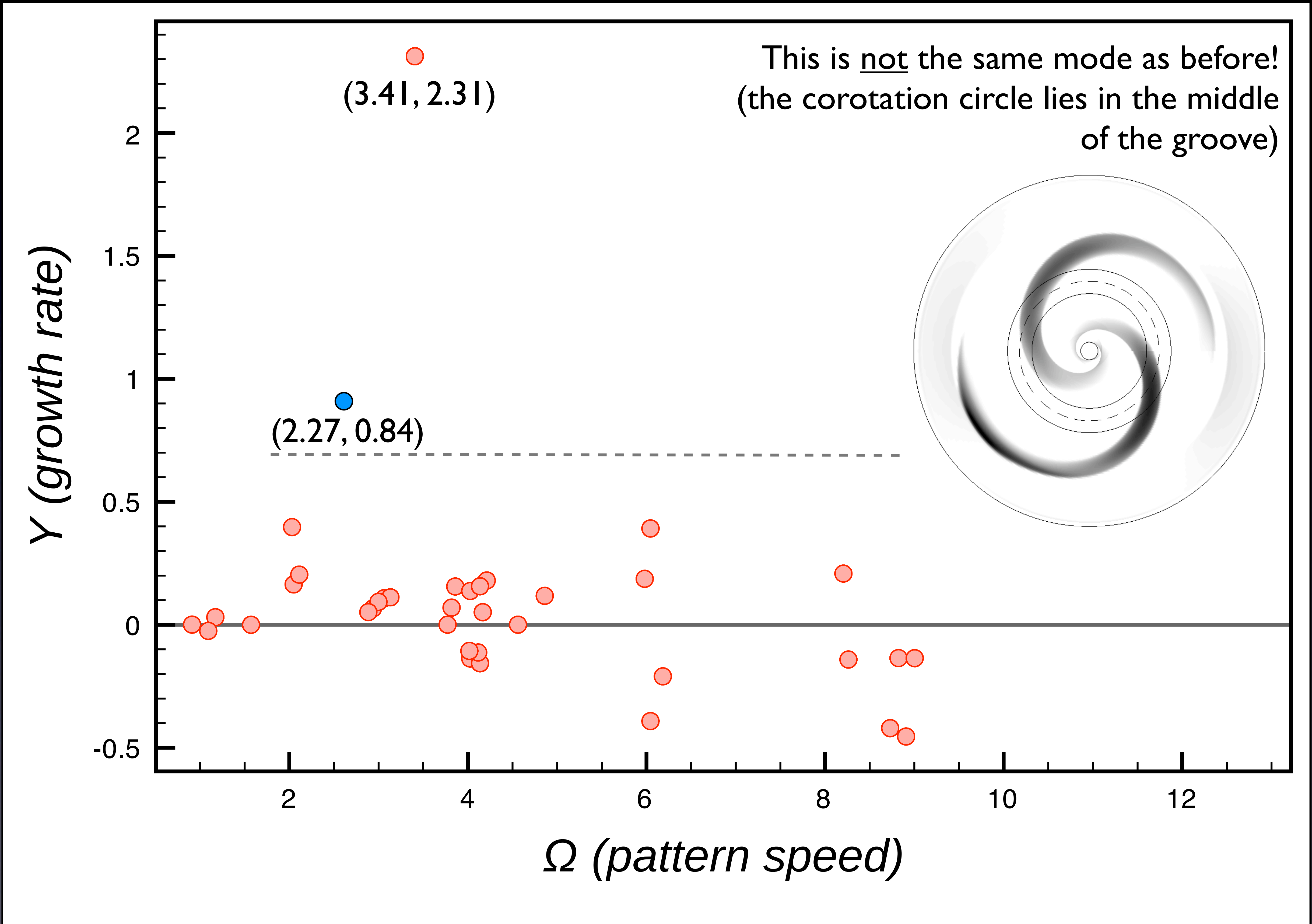
log[σ - Σ]

- - - Corotation

σ

Disk with groove, q_D = 0.63



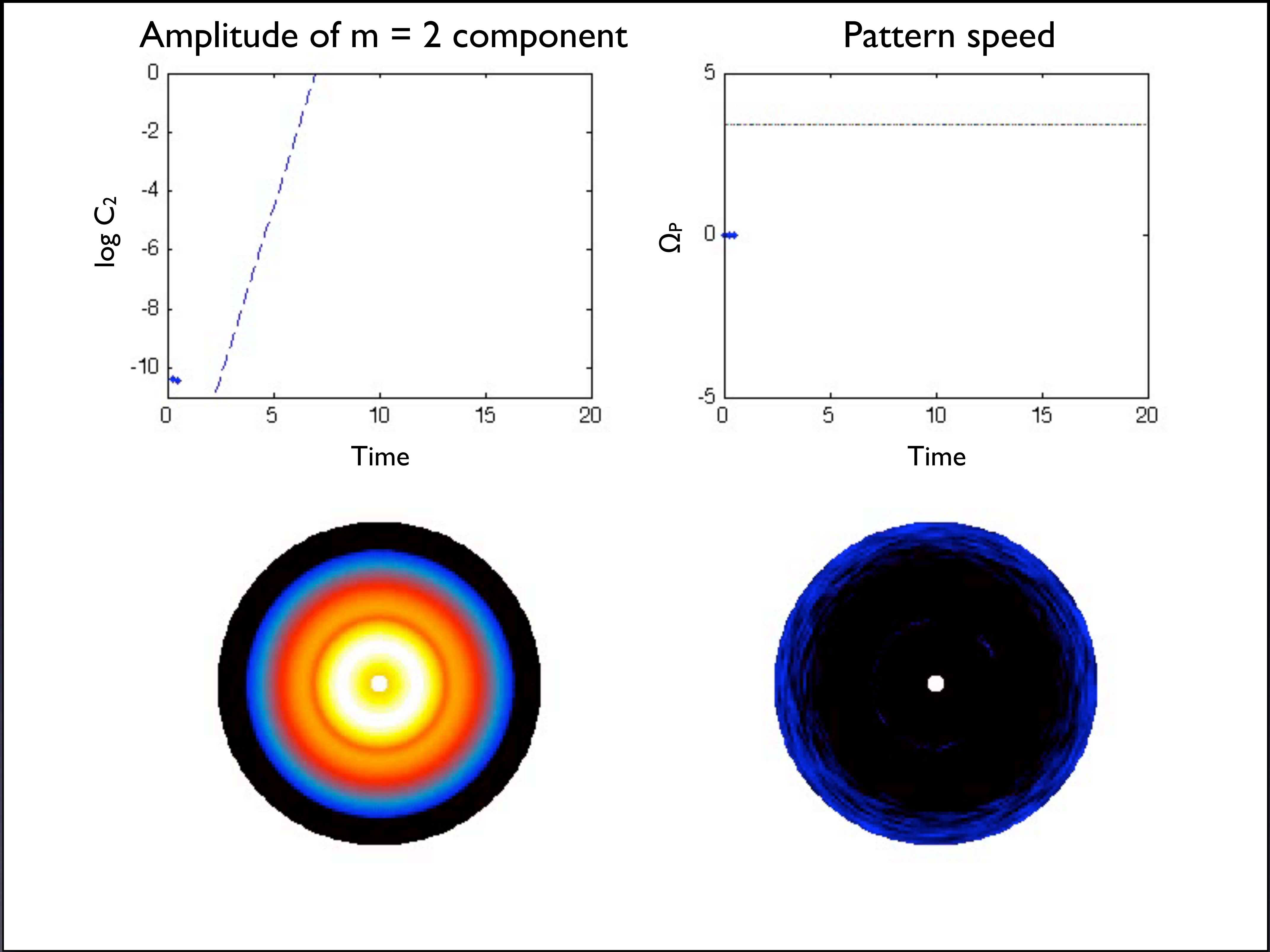

This is not the same mode as before!
(the corotation circle lies in the middle of the groove)
(3.41, 2.31)
(2.27, 0.84)
Y (growth rate)
Ω (pattern speed)
2
1.5
1
0.5
0
-0.5
2
4
6
8
10
12

log C_2

Ω_P

Time

Time


Amplitude of m = 2 component
log C2
0
-2
-4
-6
-8
-10
0
5
10
15
20
Time
Pattern speed
ΩP
5
0
-5
0
5
10
15
20
Time

All disk models

High mass
q_D = 0.63

no groove

Low mass
q_D = 0.08

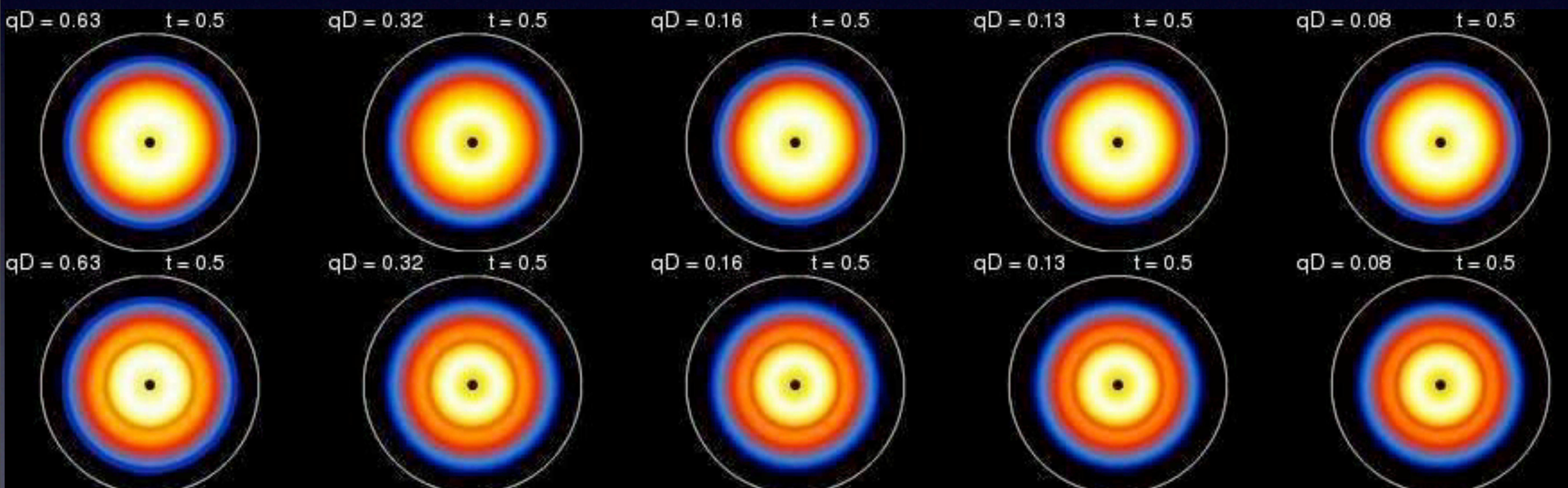
with groove

All disk models

High mass
q_D = 0.63

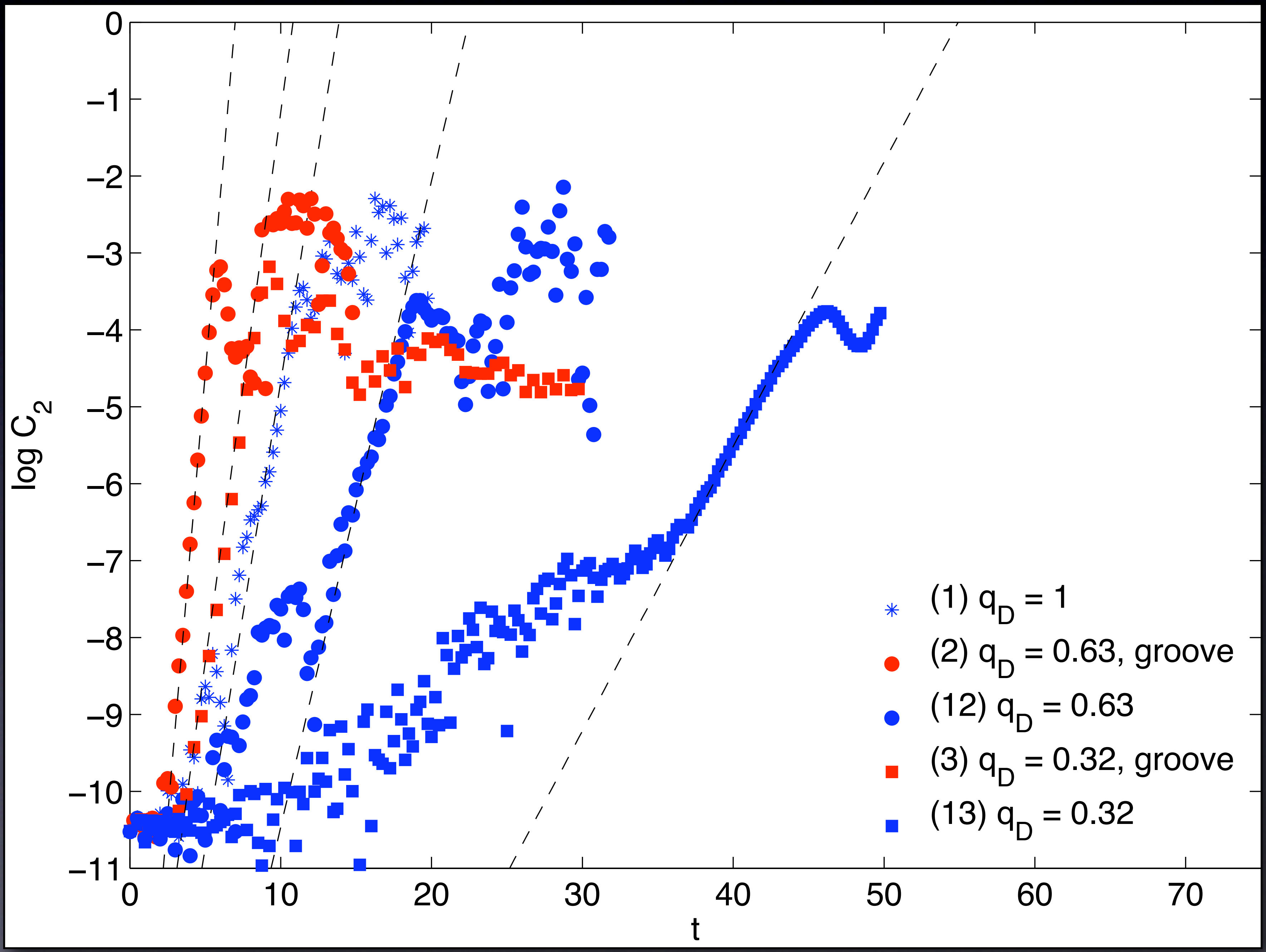
no groove

Low mass
q_D = 0.08



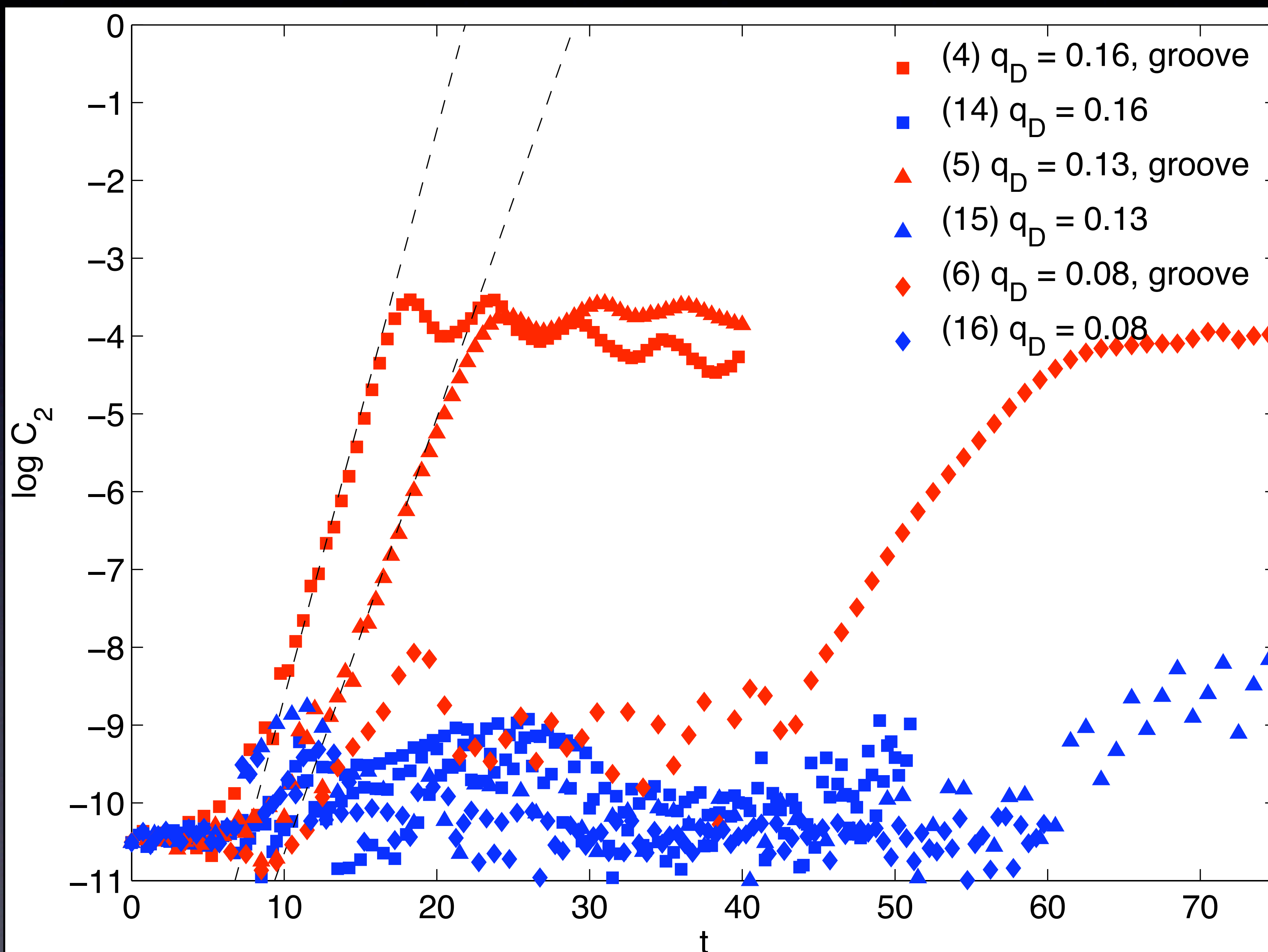
with groove

Instability growth [high q_D]



Groove mode outpaces the growth of the "base" disk

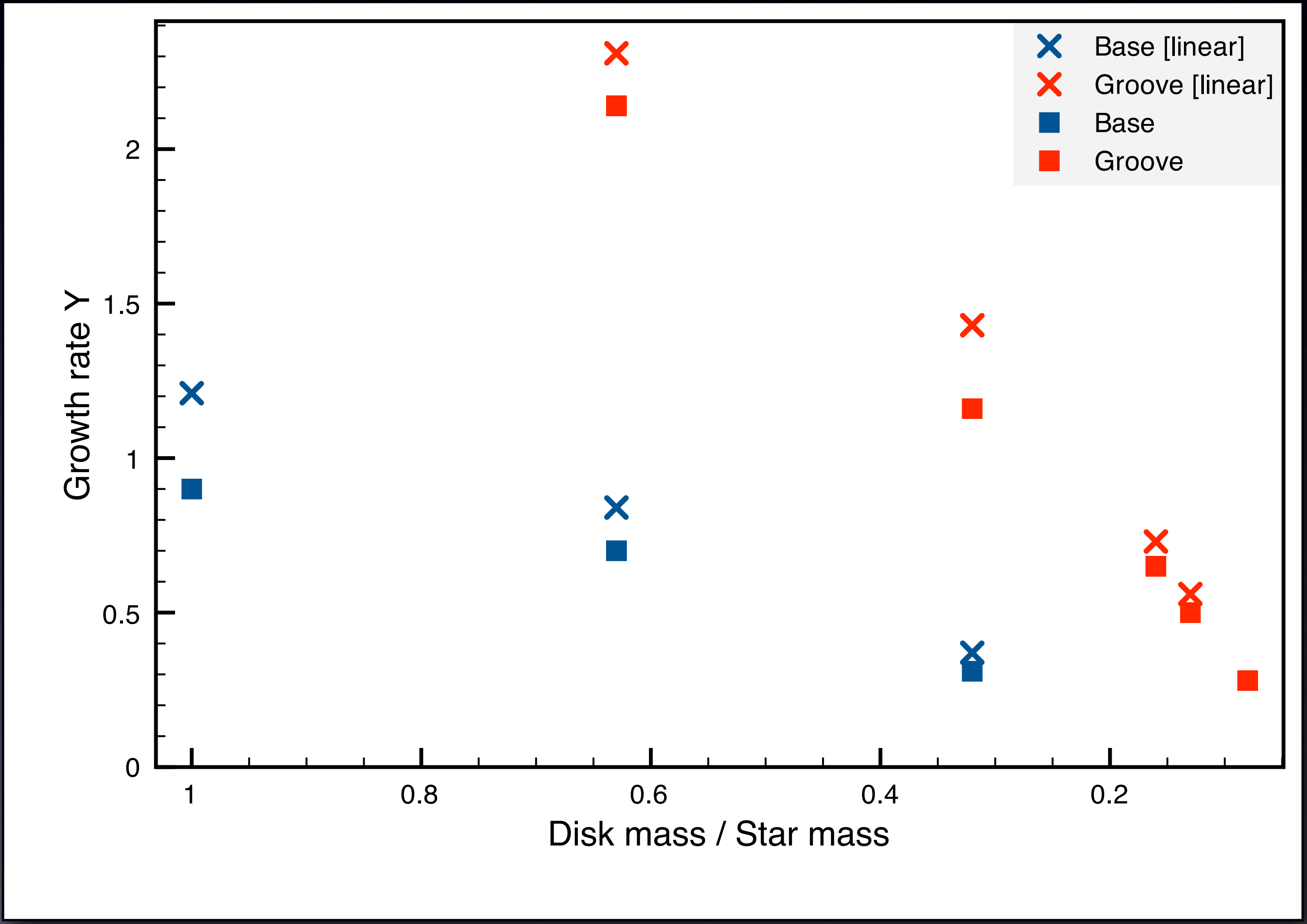
Instability growth [low q_D]



Base models cease to be unstable at q_D = 0.32.

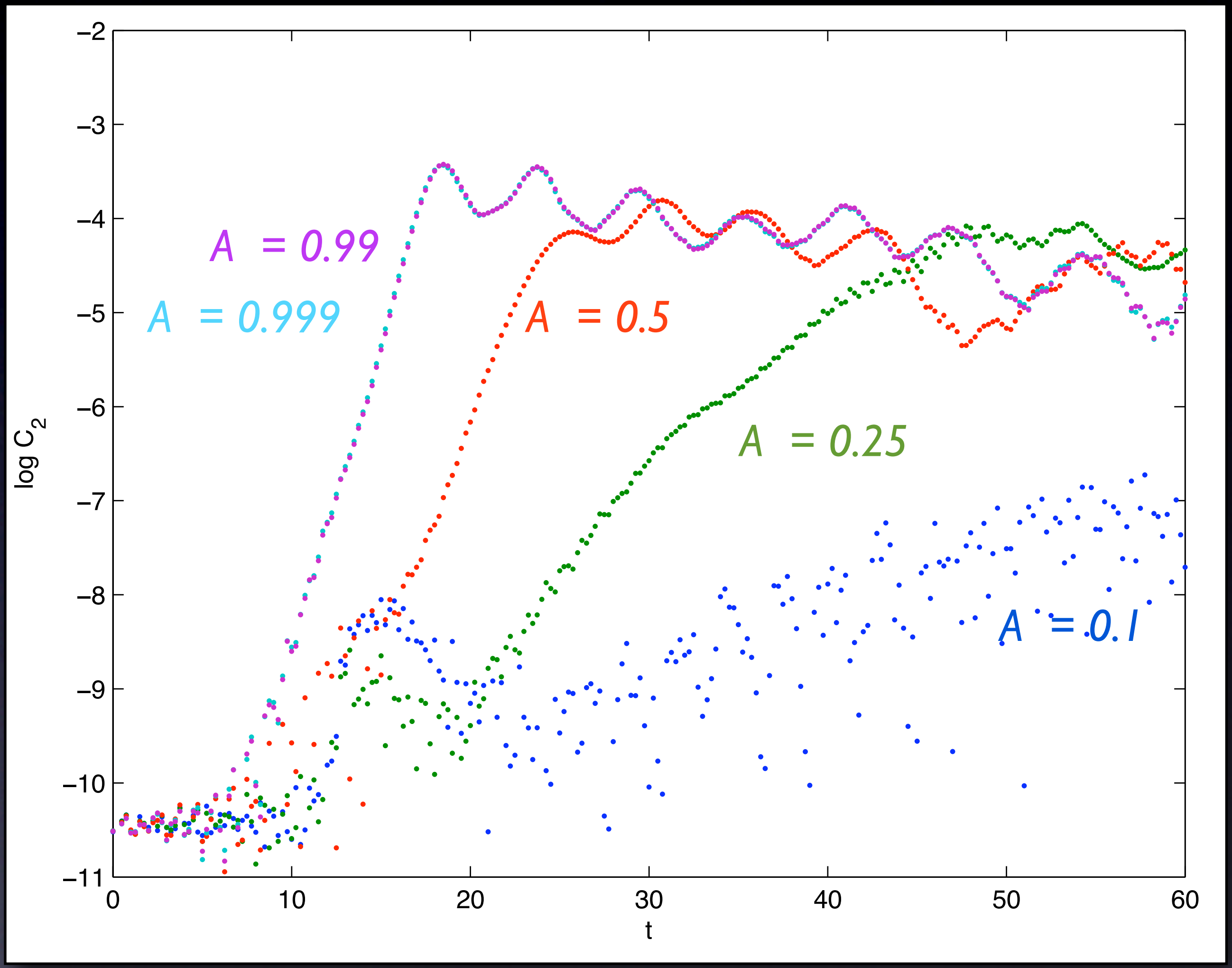
Models with a groove are unstable down to q_D = 0.08 (Q = 2.76)!

Growth rates

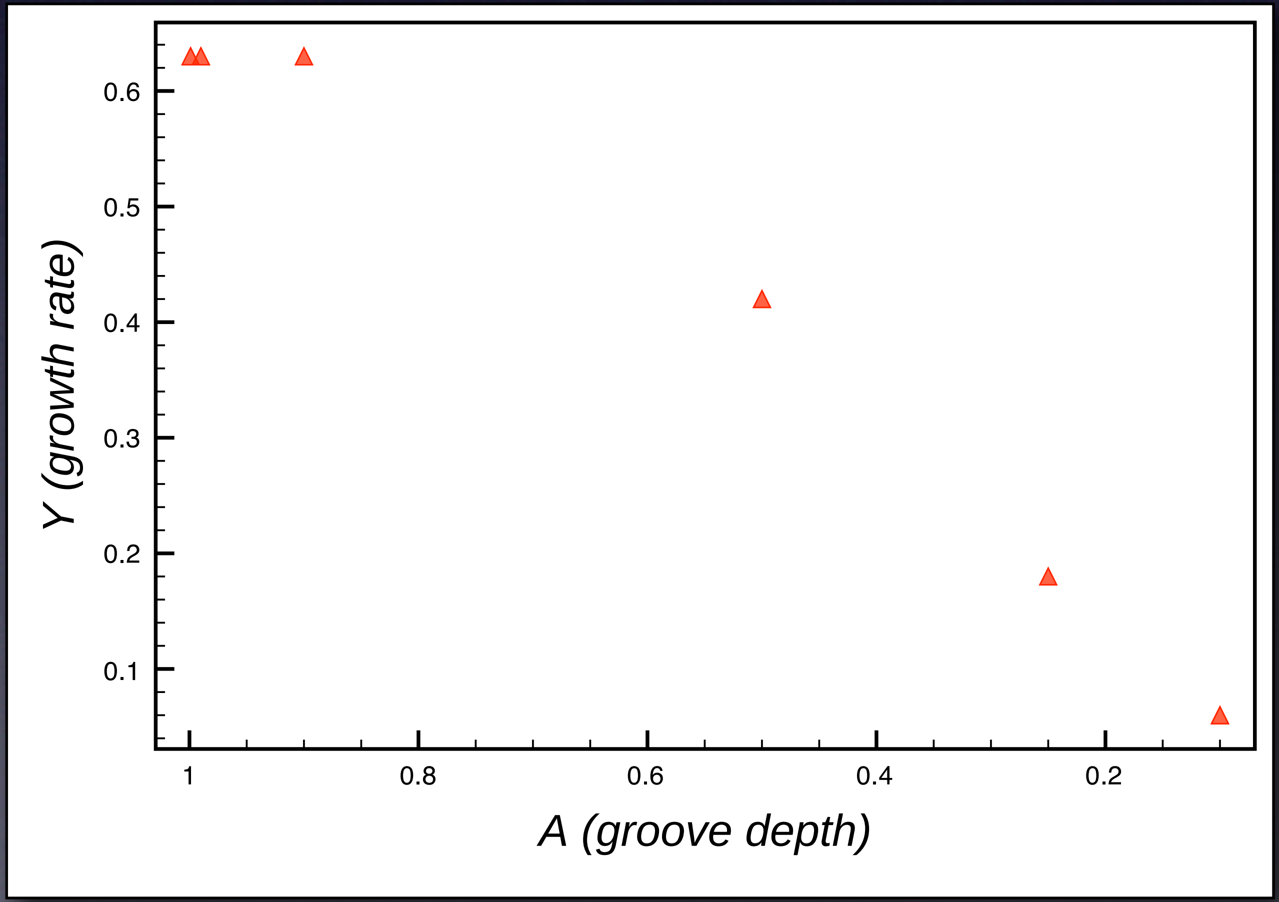


- Disk models with a groove are unstable down to $q_D = 0.08$; lower masses do not show resolved instabilities in the hydro simulations and have noisy EV spectra in mode analysis
- We believe the disks might be unstable at even lower masses, but our hydro code is too dissipative.

Dependence on depth



- The growth rate depends on the depth of the groove, but levels off at density depression of ~80%



Gap filling

The net effect of the non-axisymmetric instabilities is to redistribute disk mass and angular momentum.

Tendency to *suppress* the gap!

SI act similarly to an additional source of viscosity.

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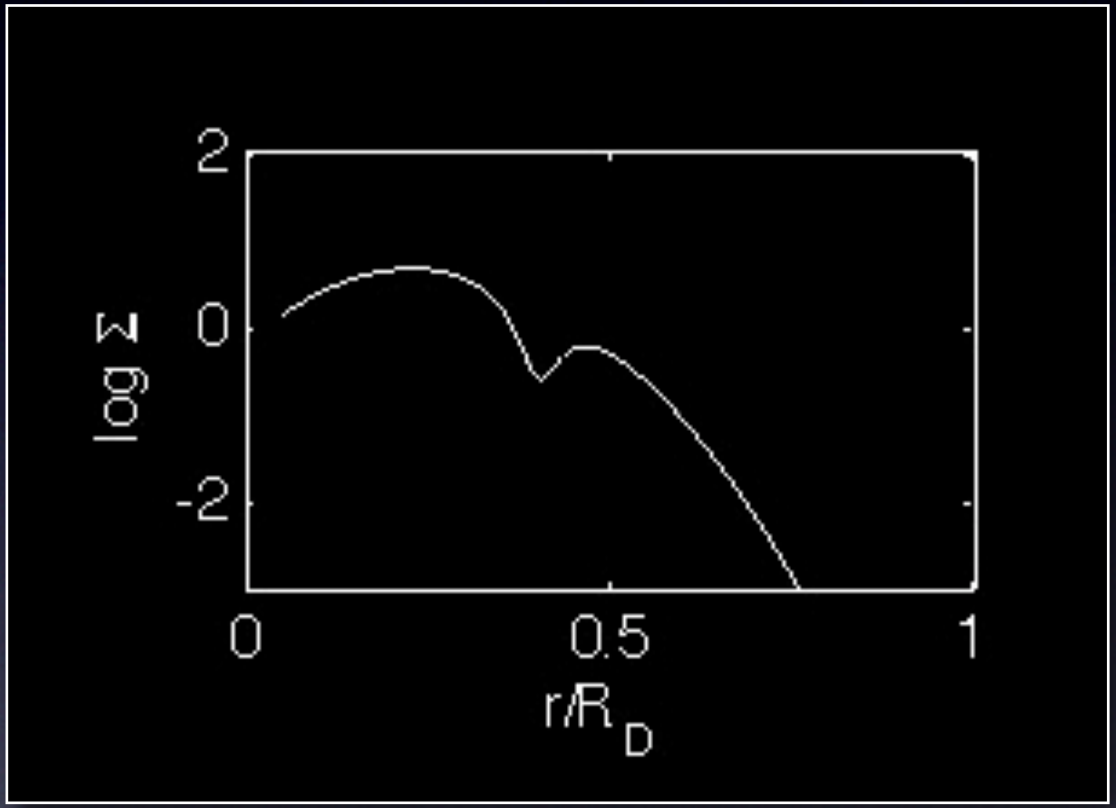
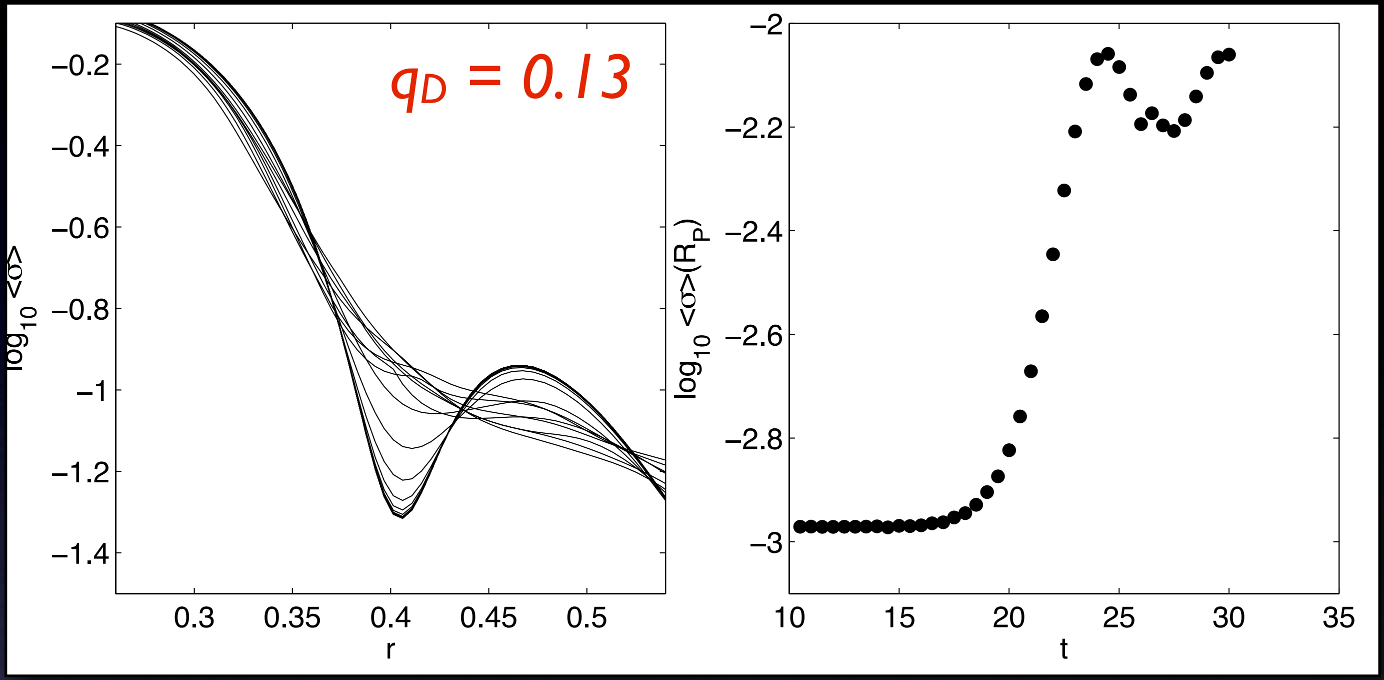
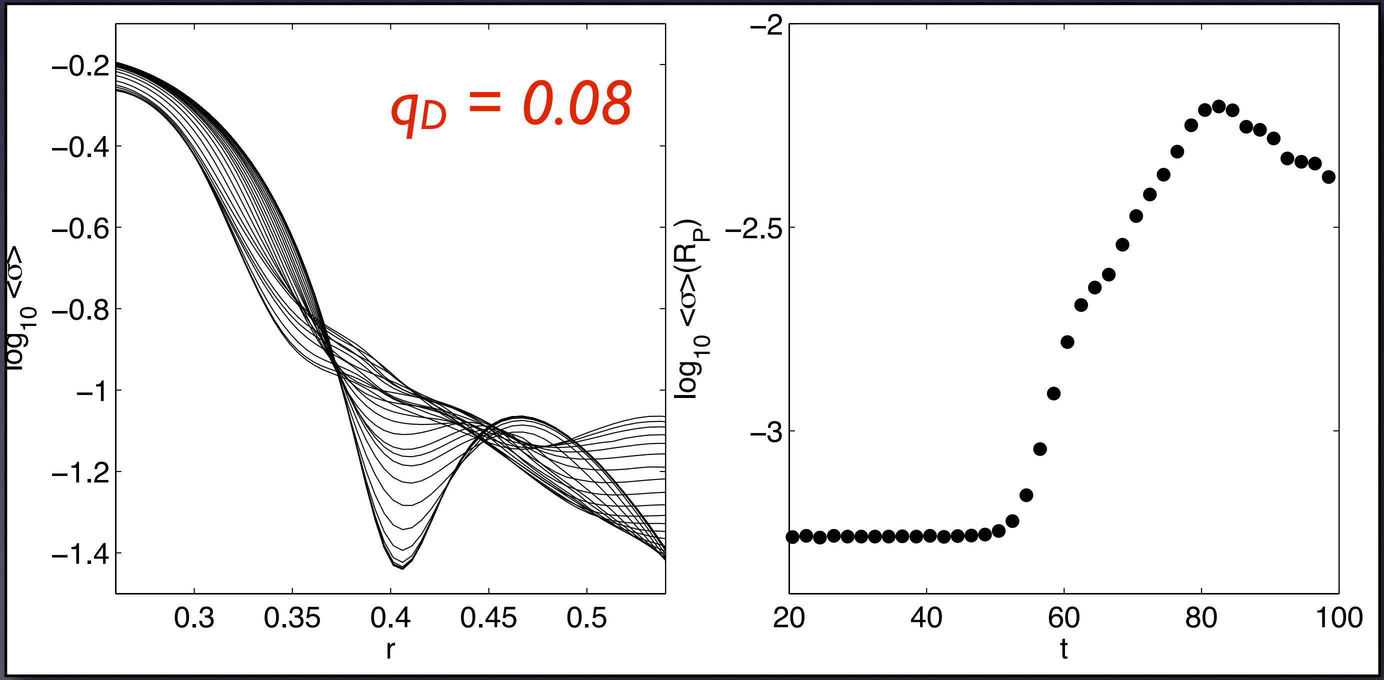


Tendency to *suppress* the gap!

SI act similarly to an additional source of viscosity.

Gap filling





Once the spiral pattern is estabilished, the gap quickly gets filled.

$$t_{fill} \sim \frac{\Delta r^2}{\nu}$$

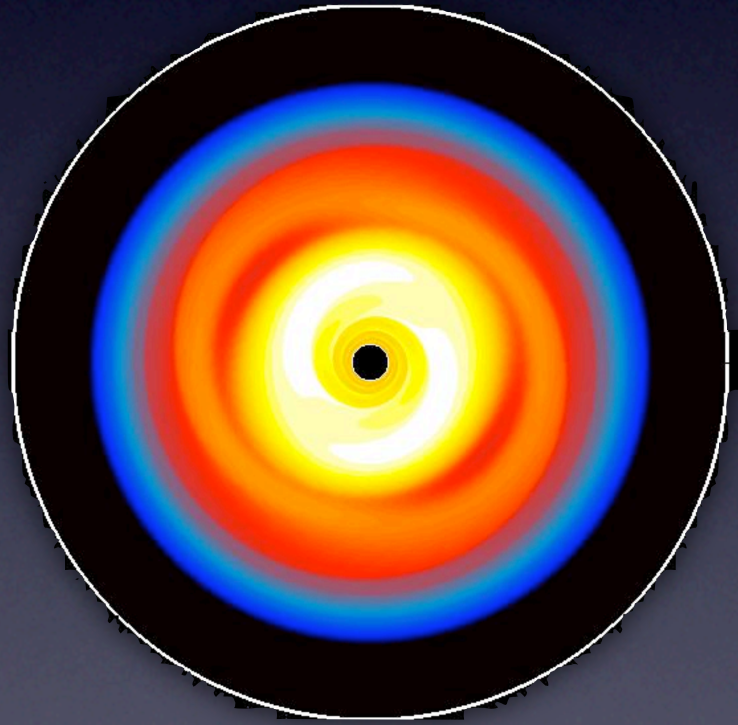
Estimate an order-of-magnitude *effective* α

Gap filling

$$t_{fill} \sim \frac{\Delta r^2}{\alpha c_s H}$$

Get $\alpha \approx 0.13$ for $q_D = 0.13$
$\alpha \approx 0.06$ for $q_D = 0.08$

➡ This class of spiral instabilities can provide a *huge* source of viscosity!



➡ This source of additional viscosity would modify the gap-opening criterion...

Type I - II transition mass

$$\frac{M_P}{M_*} \approx 2\alpha^{2/3} \left(\frac{M_*}{\Sigma r^2}\right)^{1/3} \left(\frac{c_s}{r\Omega}\right)^3$$

$\alpha \approx 10^{-3}$ $M_P \approx 4.5 M_\oplus [0.01 M_J]$

$\alpha \approx 10^{-2}$ $M_P \approx 20 M_\oplus [0.06 M_J]$

$\alpha \approx 0.05 - 0.1$ $M_P \approx 60 - 95 M_\oplus [0.2 - 0.3 M_J]$

Signicant modification of the transition mass!

This is just a simplified estimation: the growth rate depends on the groove depth...

Conclusions

- We have studied a number of disk models through mode analysis and hydro simulations.
- Massive disk models possessing a groove in surface density were grossly unstable on a few dynamical timescales.
- Moderate mass groovy disks show a fast growing mode at masses down a factor of 4 with respect to disks without a groove.
- The instability yields a very high effective viscosity; can close the gap.
- We will improve and confirm the results obtained by running a self-consistent simulation including the formation of the gap.

Future

- The potential of the planet is not negligible, which will make it more complicate to separate spiral wakes launched by the planet and intrinsic modes of the disk
- More general density profiles and EOS will be unstable to a spectrum of interacting spiral modes
- *We need to follow the formation of the gap with a high-resolution, ab-initio hydro simulations with self-gravity.*

Thank you!

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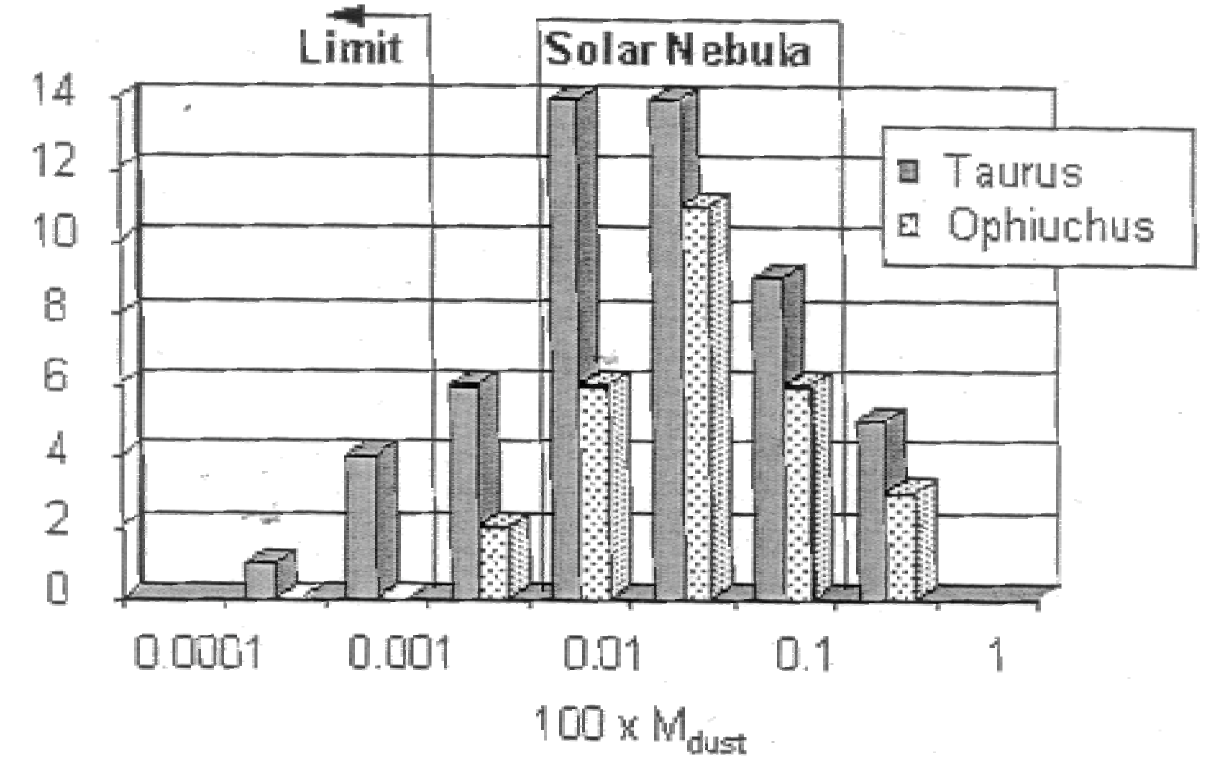
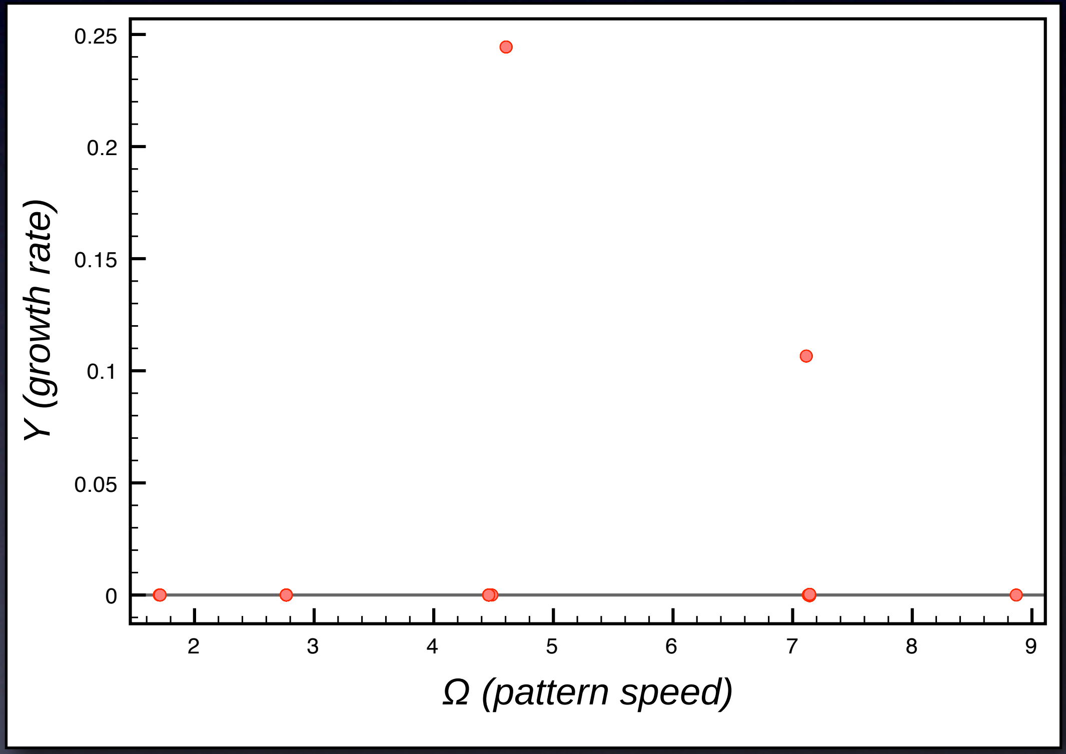
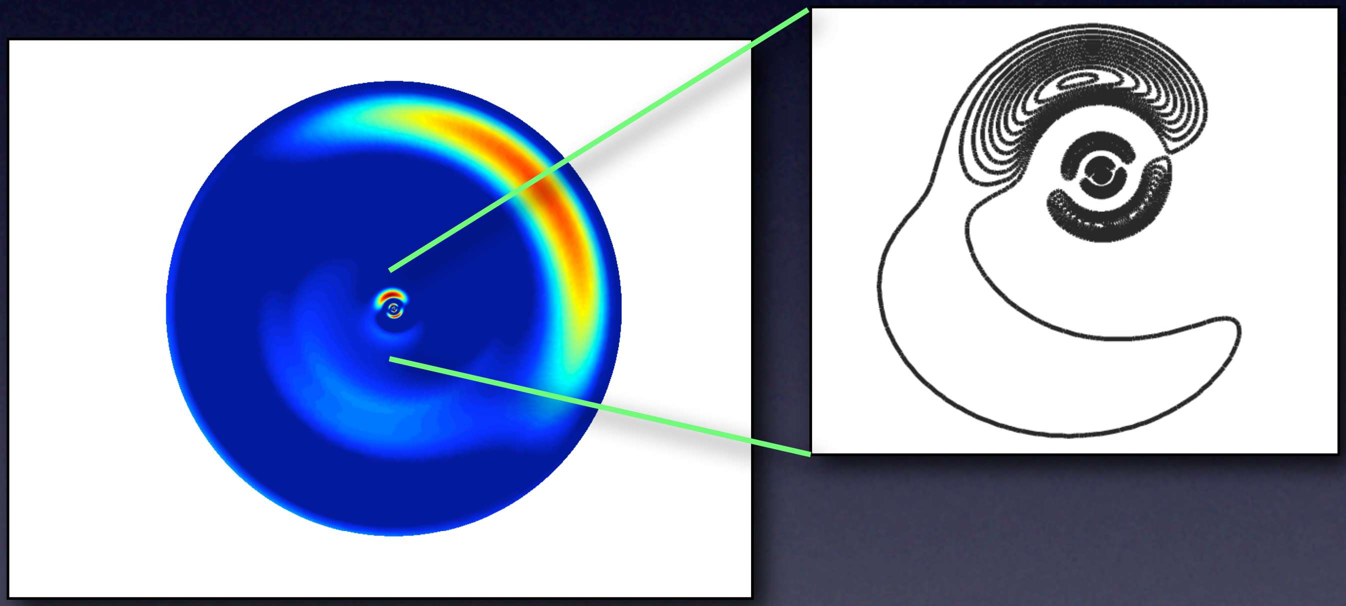


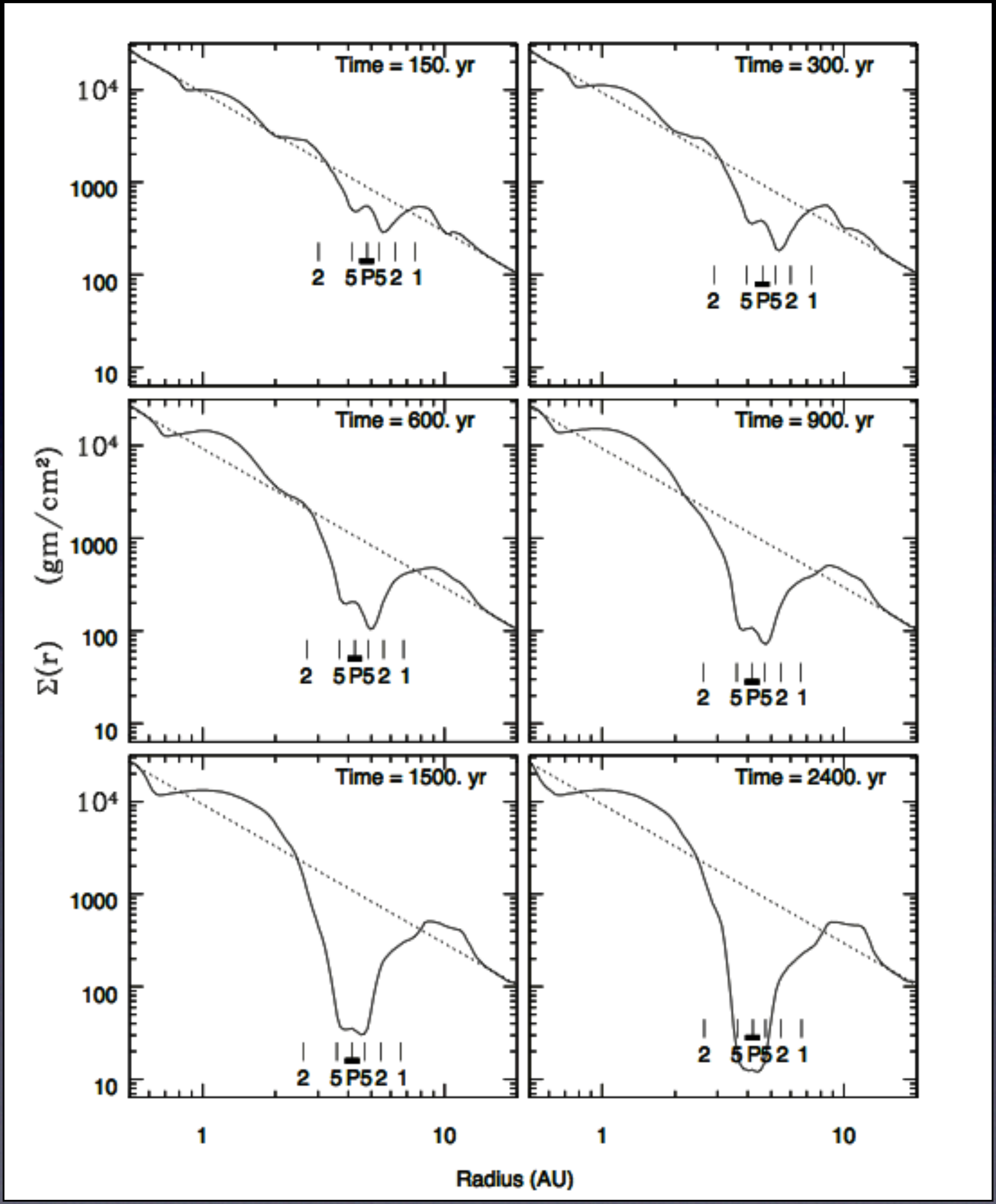
Figure 11. The histograms show the distribution of disk masses among stars in the Taurus and Ophiuchus star forming regions determined by Beckwith et al. (1990)and André et al. (1994).

Eigenvalue spectrum

Our code reproduces the eigenvalue spectrum of the ARS model






Time = 150. yr
Time = 300. yr
Time = 600. yr
Time = 900. yr
Time = 1500. yr
Time = 2400. yr
2 5 P 5 2 1
10^4
1000
100
10
Σ(r) (gm/cm²)
1
10
Radius (AU)

Density splitting

$$\Psi(R) = -\int_0^\infty \sigma(\rho)\rho d\rho \int_0^{2\pi} \frac{d\varphi}{\sqrt{R+\rho-2R\rho\cos\varphi}}$$

$$\Psi(R) = -4\int_0^\infty K\left(\sqrt{\frac{4R\rho}{(R+\rho)^2}}\right)\frac{\sigma(\rho)\rho d\rho}{R+\rho}$$

Pierens & Hure' 2005